COMPONENTS OF RISK

FOR

INVESTMENT TRUSTS

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This work is dedicated to the memory of Leslie Harold Adams M.B.E (1912 - 1993)

An active fund manager
must manage risk as
well as reward

Robin Angus[1]

[1] Haec Olim: exploring the world of investment trusts, 1981-1991

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DECLARATION

Except where specific reference is made to other sources, the work presented in this thesis is the original work of the author. The work has not been submitted, in whole or in part, for any other degree.

Andrew T. Adams

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ABSTRACT

This thesis examines various aspects of risk for UK investment trusts from the shareholders' viewpoint.

For conventional trusts, a model is developed which splits the variance of returns to shareholders into three components - variance of net asset value (NAV) returns, variance of discount returns and twice the covariance between NAV returns and discount returns. Using historical data, the relative importance of each of these components is estimated for different return intervals and for different periods of observation. There is clear evidence of excess volatility of trust share returns compared with NAV returns. Since Big Bang in 1986, there has been a significant 'double whammy' effect, meaning that discounts tend to widen when NAVs fall and narrow when NAVs rise. Overall, the results contradict the efficient market model but are consistent with the noise trader model.

Discount volatility is generally an important component of total risk for conventional trusts using monthly returns but there is considerable cross-sectional variation in the magnitude of this discount volatility. These are interesting aspects of the closed-end fund discount puzzle which have received little attention in the literature, and a cross-sectional analysis is carried out to explain the variation in discount volatility across the sector. The results suggest that the main drivers of discount volatility are new information hitting the market for trust shares and volatility of NAV returns. Discount arbitrage traders try to take advantage of discount anomalies but their activities are restricted, particularly for less marketable trusts. There is no evidence that either individual investor sentiment or UK specific sentiment has any impact on discount volatility.

Statistical measures of risk based on historical data are useful tools for conventional trust securities but are of limited use for split capital trust securities. Analysts are often more concerned with the sensitivity of these securities to changes in the underlying fundamental variables and an alternative approach to the risk assessment of split capital trusts is proposed. By differentiating discounted cash flow valuation

formulae with respect to the underlying fundamental variables, 'sensitivity measures' are derived for most split capital securities. These measures show how the present value of expected future cash flows will vary as the real discount rate changes, the real growth of income or capital value of the underlying fund changes and the estimated rate of inflation changes.

CHAPTER 1 - INTRODUCTION

This chapter sets out the objectives of the thesis, provides an introduction to the investment trust industry, outlines the structure of the thesis and provides an overview of the remaining chapters.

1.1 OBJECTIVES OF THE THESIS

The objectives of this thesis are:

(i) to identify, from the shareholders' viewpoint, the main components of risk for UK investment trusts;

(ii) to quantify the importance of these components;

(iii) to discuss the theoretical and practical implications of the findings.

Although perceptions of risk vary, the volatility of a trust's share performance is widely accepted as an important measure of risk from the shareholders' viewpoint and is adopted as the measure of risk in this thesis. Share beta, which measures the sensitivity of the trust share's return to the return on the market, suffers from the problem of having to choose an appropriate benchmark index and is rarely used by investment trust practitioners.

1.2 THE INVESTMENT TRUST INDUSTRY

Investment trusts are UK public companies whose assets consist of a portfolio of shares or other securities. They enable investors to purchase an interest in a professionally managed fund. 'Generalist' investment trusts combine investment flexibility with the opportunity to diversify by spreading investments over several markets and sectors. 'Specialist' investment trusts provide a vehicle for investment in some specialist area such as a particular geographical region or a specific sector of industry. 'Split capital' investment trusts (splits) have innovative capital structures which attempt to match the risk, income and tax preferences of different types of potential investor.

Investment trusts are subject to the regulation of the Companies Acts and the London Stock Exchange. In addition, to secure certain tax concessions, investment trusts seek approval from the Board of the Inland Revenue for each accounting period[1] (see Adams, 1989). Investment trusts themselves are not regulated by the Financial Services Act 1986 although their savings schemes are subject to this legislation.

Ultimate responsibility for running the affairs of an investment trust lies with the board of directors, but day-to-day administration and investment management is normally delegated to professional investment managers. These investment managers will usually be members of a management group rather than directly employed by the investment trust. The group may also manage other investment trusts together with other types of fund such as pension funds or unit trusts.

In common with any other company, an investment trust has a fixed (or 'closed') capital structure which must contain share capital but which may also include loan capital. The life of an investment trust normally starts with a new issue of ordinary shares (see Levis and Thomas, 1995). The number of ordinary shares then remains fixed apart from subsequent share issues, the most common type of issue nowadays being the C share issue[2] (see Adams and Szakacs, 1996). To liquidate their holdings, investors must normally sell their shares to other investors.[3] An advantage of this 'closed-end' structure is that the fund managers can act in the best long-term interests of their shareholders without having to worry about a possible reduction in the

[1] Approval will not be granted to an investment trust unless *inter alia*: it is resident in the UK for tax purposes; its income is derived wholly or mainly from shares and securities; no holding in any one company represents more than 15% by value of its investments; its ordinary share capital is listed on the Stock Exchange; its Memorandum or Articles of Association prohibit the distribution by way of dividend, of surpluses arising from the realisation of investments; it does not retain more than 15% of the income it derives from shares and securities.

[2] The new money raised by a C share issue is kept separate from the investment trust's existing funds until a specified percentage (e.g 80%) has been invested, at which point the new pool of money ceases to be run independently and its shares (the C shares) are converted into ordinary shares on an NAV basis.

[3] However, a number of trusts have a limited life. There may be a fixed redemption date but very often there are a number of optional winding up dates. Furthermore, 'buy backs' are becoming popular (see Section 2.2).

underlying portfolio of assets in adverse market conditions, as is the case with unit trusts.[4]

Investment trusts pay corporation tax on unfranked income[5] after deduction of management expenses and loan interest paid; witholding tax on income from overseas investments may be offset against UK tax liabilities provided there is a double taxation agreement. Franked investment income may be passed on to shareholders without incurring further tax.[6] If approved by the Inland Revenue, an investment trust fund is exempt from tax on capital gains (Finance Act 1980, s81). In practice, it is unusual for investment trusts to pay much corporation tax, apart from a few overseas specialists.

Investors in an investment trust are taxed on the same basis as for investment in the securities of any other type of company. This means that for some institutional shareholders, contingent capital gains tax liabilities may be an important factor restricting the sale of investment trust holdings. Private investors often hold investment trust shares under a Personal Equity Plan (PEP) which, subject to certain restrictions, means that both income and capital gains are tax-free.

As the ordinary shares of an investment trust must be listed on the Stock Exchange, the procedure for dealing in the shares is the same as for other listed shares. So investors who wish to buy or sell investment trust shares do so at a price that reflects the supply and demand for the shares, rather than the underlying net assets of the company. Nevertheless, investors generally regard conventional[7] investment trust shares as essentially claims on assets, and investment trust analysts watch the relationship between the price of the investment trust shares and the value of the underlying net assets very carefully.

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[4] Unit trusts are 'open ended' so that if a sufficient number of unitholders wish to sell their units, it may be necessary to sell assets in the underlying portfolio to generate sufficient cash to meet the demand. Unit trust prices are determined by a fixed 'break up' formula.

[5] The term 'unfranked income' includes all income received other than UK dividend income.

[6] Although not taxable, the franked investment income is reported gross in the investment trust's accounts with a provision made for the total of tax credits associated with dividends received. If the franked investment income reported in the accounts is greater than the dividends the trust pays to its own shareholders, the balance of franked investment income is carried forward and set against dividends for the following year.

[7] Conventional investment trusts are those without a split capital structure (see 2.3).

The net asset value (NAV) of a conventional investment trust is obtained by deducting prior capital[8] from the value of underlying assets, and is normally expressed on a per share basis.[9] Discount to NAV is then defined as NAV less share price, expressed as a percentage of NAV. Discounts vary widely from one trust to another and also vary over time, but attempts to explain these variations have met with no more than limited success. Sometimes analysts talk of the sector average discount. This is a weighted average of discounts across the conventional investment trust sector, where the weights are the market capitalisations of the individual trusts. Some analysts argue that the concept of a sector average discount is less valid nowadays given the high degree of specialisation within the industry.

The management of an investment trust will attempt to 'add value' by skilful portfolio management, and the benefits and costs of the professional management will be reflected in the NAV performance. In measuring the investment performance of the fund managers themselves, the correct approach is therefore to use underlying NAV rather than the market value of the shares.[10] But the actual return obtained by an investor in the investment trust shares will include the effects of a change in discount ('discount return') as well as NAV return.

Discounts are important in the context of investment trust risk analysis. As part of the return from a conventional investment trust share is due to changes in the discount, discount variation over time contributes to the variance of returns from investment trust shares. Furthermore, discount changes over a period may be related to returns from the underlying net assets of the trust over the same period. If there is positive correlation between discount returns and NAV returns (i.e. a tendency for discounts to narrow when NAVs rise), this will increase the variance of share returns. These are interesting aspects of the discount puzzle that have received little attention in the literature.

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[8] Prior capital is normally deducted at nominal value.

[9] Complications can arise in calculating NAV when there are warrants (or convertibles) in issue - see 4.4.2.

[10] Specialist trusts' performance should be measured against appropriate benchmarks. The managers are responsible only for performance within the specialist area, the choice of area having been made by the investors.

The investment trust is not the only type of UK corporate investment vehicle that trades at a variable price relative to its underlying net assets. For example, property investment and development companies normally trade at a discount to NAV. As with investment trusts, the discount varies widely from one company to another within the sector and the sector average discount varies over time (see Adams and Venmore-Rowland, 1990).

Over one-tenth of the investment trust sector by value consists of split capital investment trusts (splits) which is a sub-sector distinct from conventional investment trusts. Each split trust has more than one main class of share capital and is designed to meet simultaneously the needs of different types of investor. Expected cash flows for a class of share capital may be defined in nominal terms or may depend on the income growth or capital growth of the underlying fund, which is generally invested in the UK. Split securities are normally assessed by considering the profile of expected cash flows and not with reference directly to underlying assets.

1.3 STRUCTURE OF THE THESIS

This thesis is essentially made up of three parts. Chapters 2 and 3 provide the necessary institutional and academic background for the later research chapters. Chapters 4 and 5 adopt a statistical approach based on historical data to analyse the risk of conventional investment trusts. One would expect that the volatility of the NAV return and the volatility of the trust share return to be equal. Minor factors have been suggested to explain any differences but these seem incapable of explaining observed differences in the US. It is interesting to see if this is the case in the UK. Chapter 6 then analyses the sensitivity of split capital investment trust securities to changes in the underlying fundamental variables using a mathematical rather than a statistical approach. The restricted life of these trusts means that the risk profile of split securities changes over time and a statistical approach is therefore of limited use.

1.4 CONTENTS OF SUBSEQUENT CHAPTERS

Chapter 2 provides the institutional background for the thesis. It gives a history of the investment trust industry, outlines the different types of investment trust security and introduces the discount to net asset value puzzle. US closed-end funds are described and compared with UK investment trusts. Risk assessment methods employed by practitioners for both conventional and split capital investment trusts are then examined.

Chapter 3 reviews the existing literature relating to efficient markets, asset pricing and closed-end funds (including conventional investment trusts). This includes work on topics relating to the excess volatility of investment trust shares and the factors that influence discount volatility. Market efficiency and its relevance for this study is examined. Most of the research in this area concerns US closed-end funds rather than UK investment trusts. The chapter contains predictions from the literature which will be investigated in later chapters.

Chapter 4 investigates the importance of the three components of the variance of share returns for conventional investment trusts with a view to assessing the importance of discount volatility and whether investment trust shares tend to underreact or overreact to changes in NAV. This has implications for efficient markets and noise trader theories. The analysis is carried out using different return intervals and over different periods of observation. The results are compared and contrasted with those of other published research, which generally relate to US closed-end funds.

Chapter 5 identifies possible trust attributes that may influence the cross-sectional variation in discount volatility for conventional trusts, where discount volatility is defined as the square root of the variance of monthly discount returns. Regressions are then carried out to explain the cross-sectional variation in discount volatility using the identified trust attributes as explanatory variables. The linkages between the results of the regression analysis and the results of both Chapter 4 and previous research are discussed. In particular, the implications of the results for possible explanations of the discount volatility phenomenon are assessed.

Chapter 6 develops simple valuation models for split capital trusts and discusses how the fundamental variables which determine the present values should be estimated. Recent extensions to the concept of equity duration ('sensitivity measures') are also discussed. Sensitivity measures for split securities are then calculated with respect to each of the underlying fundamental variables and the practical importance of the work is discussed.

Chapter 7 summarises the main conclusions of the thesis. The contribution of each of the chapters is reviewed and the results qualified by identifying shortcomings in the models employed. Finally, future research questions raised by the thesis are identified.

CHAPTER 2 - INSTITUTIONAL BACKGROUND

2.1 INTRODUCTION

This chapter provides the institutional background required for later chapters, in particular industry perspectives of risk. Some of the information is derived from interviews with practitioners (see acknowledgements). The historical development of the UK investment trust industry is first examined in some detail to aid understanding of the industry as it stands today. The discussion is extended to include US closed-end funds as most of the relevant academic research discussed in Chapter 3 relates to these vehicles.

2.2 HISTORY OF THE INVESTMENT TRUST INDUSTRY

The origins of the investment trust industry can be traced back to the launch of Foreign & Colonial Government Trust in 1868 (McKendrick and Newlands, 1999). The aims of this trust, set out in its prospectus, were:

> "to give the investor of moderate means the same advantages as the large capitalist, in diminishing the risk of investing in Foreign and Colonial Government stocks by spreading the investment over a large number of stocks."

Before this issue, a straightforward and easily accessible method of gaining exposure to a spread of stocks simply did not exist for 'the investor of moderate means'. The only credible alternative was considered to be 'The Funds' (British Government stock), typically yielding about 3% while overseas government bonds generally offered over twice as much (Fredman and Scott, 1991).

Great Britain was the dominant economy in the world around the time of the Foreign & Colonial issue, generating large amounts of investment capital, and emerging economies, especially the US, were keen to encourage inward investment. There was particular interest in US investment amongst Scottish investors, and the Scottish American Investment Trust was formed in Dundee in February 1873. This was partly modelled on the Foreign & Colonial Government Trust and was remarkable in that Dundee was not a financial centre. Then two months later in April 1873, the Scottish

American Investment Company Ltd was formed in Edinburgh. This was constituted as a company rather than a trust and was the first investment trust *company* in Scotland (Newlands, 1997).

By the beginning of 1875, a total of 20 trusts had been launched and listed on the Stock Exchange, 11 as unregistered common law trusts and 9 as investment trust companies (Scratchley, 1875). Then in 1878 the unregistered common law trusts were declared illegal (Sykes v Beadon, 1878) and had either to restructure and become investment trust companies registered under the Joint Stock Companies Acts of 1862 and 1867 or had to be liquidated.

The remaining years of the nineteenth century saw continued growth of the investment trust industry, with the fixed-interest securities of US railroad companies and US land mortgages particularly popular investments for the companies. By 1900, there were 58 investment trust companies with a total market capitalisation estimated at £60m to £70m and generally invested mainly in fixed-income securities. There followed a period of further steady growth so that by the beginning of World War I, there were 90 trusts, 58 of them English and 32 Scottish (Burton and Corner, 1968).

Following the introduction of corporation profits tax in 1921, many investment trusts made their first moves into equity investment. This was because UK equities provided franked investment income that could be passed through to the investment trust shareholders without incurring further tax. Arnaud (1983) estimates that by the late 1920s a typical investment trust's portfolio consisted of equal proportions of debenture, equity and preference investments.

British investment trusts survived the 1929 Wall Street Crash relatively unscathed, having resisted the rush into risky investments which was so prevalent among US investors, and having sold US securities ahead of the crash in many cases. But the ensuing Great Depression was a difficult period, particularly for those trusts which reinvested too early after the initial slump and those that had remained exclusively in the mortgage business (Newlands, 1997).

A gradual recovery began after 1932 and continued until the outbreak of war in 1939. During World War II, investment trusts were required by the Defence Acts to place all US investments in the hands of the Treasury. It is estimated that around £45m of US securities were requisitioned from the investment trust sector (Burton and Corner, 1968). When dollars were needed by the British government, the securities were sold and the sterling equivalent returned to the trusts. As a result, the US proportion of investment trust portfolios were considerably reduced by the end of the war. Exchange controls were reduced in 1947, but foreign investment could only be made through an artificial 'investment dollar' which, being a scarce resource, traded at a variable premium to the official exchange rate. Nevertheless, this did not stifle major reinvestment by investment trusts in US stocks, a process that continued well into the 1950s.

The Companies Act of 1948 was significant for investment trusts because it introduced a requirement for the market value of investments to be included in the annual Report and Accounts. Thus, for the first time it was possible for analysts to calculate the NAV and hence the discount to NAV (Gammell, 1971). Prior to the 1948 Act, the true underlying value of a trust's assets was known only to those closely involved with the management and auditing of that trust.

The 1950s were buoyant years for the investment trust industry with expansion via both new issues and rights issues. The 'cult of the equity' led to significant increases in the value of underlying equities and investment trusts made use of their ability to 'gear up' using long-term debt at low rates of interest. As a consequence, NAVs rose substantially (Arnaud, 1983).

The 1965 Finance Act introduced corporation tax, long-term capital gains tax and an investment currency premium surrender tax, all of which had a negative impact on the investment trust industry (Arnaud, 1983). The new capital gains tax applied to disposal of assets after 5 April 1965, irrespective of how long the assets had been held. Capital gains were simply determined by deducting the acquisition cost of an asset from the disposal proceeds; losses were allowable against gains of the same or future years. But the 30% rate of tax on chargeable gains was generally far lower than marginal rates of tax on unearned income.[1] The investment currency premium (often referred to simply as the 'dollar premium') surrender tax required 25% of the investment currency premium on sale of non-sterling securities to be surrendered to the Bank of England (Day and Jamieson, 1975).

There was a shift in investment trust ownership from private to institutional shareholders over the 1960s and 1970s for a number of reasons. Provision for old age was moving away from the creation of private wealth towards participation in occupational pension schemes. The tax regime penalised private investors while offering tax incentives for life assurance and pension provision. Private investors were net sellers of investment trusts throughout the 1960s and 1970s.

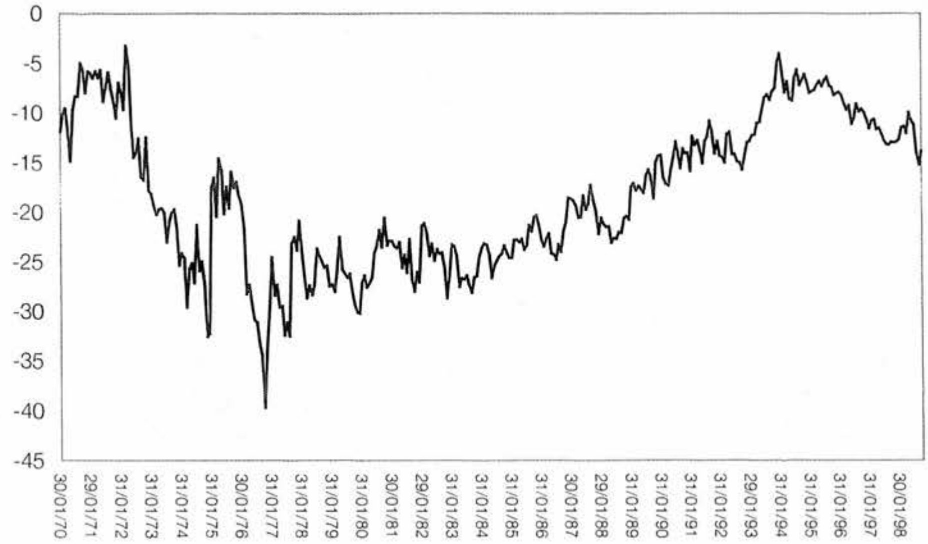
However, there was a clear need for investment trusts which catered precisely for private investors' preferences for capital gains or income, and the first split capital investment trust, Dualvest, was launched in May 1965, soon after the 1965 Finance Act. The ordinary share capital of this trust was split into two distinct categories - income shares and capital shares. The success of Dualvest led to the creation of a number of similar vehicles, which would all be known nowadays as 'traditional splits' (see 2.6.1).

The equity market boom of the early 1970s generated an upsurge in the volume of investment trust new issues. During the period 1970 to 1972, 40 new trusts were formed, adding over £500 million to the sector. By the end of 1972, total assets of the sector exceeded £8 billion (Arnaud, 1983) and a classic case of supply exceeding demand was created. The equity market peaked in 1972 and there followed a severe bear market lasting until the end of 1974. The sector average discount widened from below 10% to around 40% during the course of this bear market and remained high, oscillating around 30%, for the rest of the 1970s[2] (see Figure 2.1). Both private and institutional investors had lost interest in investment trusts. This left many trusts

[1] The highest marginal rate of tax on unearned income, including an investment income surcharge of 15%, was 98%.
[2] Discounts would be narrower than shown over the period 1970-80 if allowance were taken for the contingent investment dollar surrender tax and contingent capital gains tax.

vulnerable to take-over by an institutional investor or industrial/commercial company.[3] Several nationalised industry pension funds made successful bids for trusts. In 1977 and 1978 alone, corporate action took over 11% of the UK investment trust industry (Newlands, 1997).

Figure 2.1 - The investment trust sector average discount since 1970



Source: Datastream

Exchange controls were lifted when the Conservatives came to power in October 1979. The consequent removal of the investment currency premium initially had an adverse effect on a number of investment trusts as it reduced the portfolio valuation of overseas securities. But the 1980 Finance Act helped the sector by exempting investment trust funds from tax on capital gains. This allowed tax-exempt investors, such as pension funds and charities, to invest in investment trusts without any capital gains tax disadvantages.

The 1980s saw a gradual narrowing of the sector average discount from over 30% to below 15% (Figure 2.1). In response to the threat of takeover, a number of

[3] An institutional investor could acquire a substantial portfolio of investments at a relatively attractive price. An industrial/commercial company could offer its own shares to the investment trust shareholders and then sell off the underlying assets of the investment trust for cash, as an alternative to a 'rights' issue.

investment trusts took action to reduce or eliminate the discount by setting optional winding-up dates or by adopting a more specialised investment policy. A more drastic measure was 'unitisation', whereby the investment trust turned itself into a unit trust, which immediately removed the discount because unit prices are directly related to the value of underlying assets.[4] More generally, it is widely accepted among practitioners that the narrowing of the sector average discount during the decade was partly due to incremental demand from private investors resulting from the introduction of savings schemes and PEPs by many trusts.[5] Other possible factors included greater financial incentives for independent financial advisers to recommend investment trusts and more prominent newspaper advertisements publicising annual results.

In 1987, a conventional trust, River & Mercantile[6], was reorganised to create a traditional split capital structure, with its ordinary share capital divided into income shares and capital shares, resulting in the virtual elimination of the trust's previous 20% discount. Two other conventional trusts, River Plate & General and General Consolidated, followed suit shortly afterwards. Then later in 1987, Scottish National reorganised into a split with certain novel features. The idea was to create a more aggressive structure by the inclusion of both stepped preference shares (see 2.6.7) and zero dividend preference shares (ZDPs, see 2.6.6) as well as income shares, capital shares and warrants. This was the first time ZDPs had been issued by an investment trust. Since 1989 it has been more common to create 'quasi splits' with only two principal classes of share - ZDPs and highly geared ordinary income shares (now known as 'income & residual capital shares', see 2.6.8). Stepped preference shares have been less popular and have featured in only four trusts.

The early 1990s were characterised by a significant growth in investment trust funds due to rising equity markets and considerable new issue activity. The period also saw a further reduction in the sector average discount to reach a low of 3% at the beginning of 1994. Then followed the remarkable year of 1994 which saw a huge

[4] Although unit trusts were not created until the 1960s, the unit trust industry overtook investment trusts in terms of assets under management in the mid-1980s (Adams, 1989).
[5] Savings schemes are effectively execution-only share-dealing services, whereas PEPs are tax-efficient 'wrappers' with entry fees and management fees.

injection of new money into the sector. A total of £4.6bn was raised, excluding the £3.6bn listing of 3i. Practitioners argue that this led to an oversupply of investment trust shares and a widening of the sector average discount to reach a level of 12% by the end of 1997 (see Figure 2.1).

The conventional investment trust sector has been undergoing a period of consolidation and contraction in recent years. A number of trusts have being wound up or have undergone capital reconstruction, offering discontented shareholders the option of a cash exit. Some of the biggest trusts have been standing on sizeable discounts and therefore vulnerable to attack. Institutional shareholders resent paying investment trust management fees when they have their own in-house fund management teams, and are therefore likely to support the winding-up of trusts. In response, the AITC started a nationwide publicity drive in the autumn of 1999 based around the slogan 'its' and financed by the investment trust industry. The AITC hopes that the publicity campaign will lead to an increase in the number of shares held by private investors (AITC, 1999).

The abolition of advance corporation tax (ACT) from April 1999 opened the way for widespread share buy-backs.[7] Buy-backs increase a trust's NAV provided that the shares are purchased at a discount, since the trust is buying its own assets cheaply. In addition, by reducing the supply of shares in the market and taking out disgruntled institutional shareholders, a buy-back can arguably reduce the discount to NAV. Previously, buy-backs were subject to ACT on the amount by which the payment exceeded the shares' value when they were first issued, which made the mechanism prohibitively expensive for most of the older trusts.

With the marked fall in interest rates over recent years, the quest for income among private investors has caused something of a renaissance of the split capital investment trust sector. Zero dividend preference shares are being issued with redemption yields of over 8% per annum and income & residual capital shares offer starting yields which are unattainable elsewhere in the market. As a consequence,

[6] River & Mercantile became Fleming Income & Growth in November 1996.
[7] A share buy-back is a transaction in which a company buys its own shares and then cancels them.

£1.8bn of new money has flowed into the split capital trust sector in 1999 (Williams de Broë, 2000). However, some commentators (e.g. Cazenove & Co, 1999) warn that hurdle rates (see 2.6.9) are very aggressive for high yield portfolios in a low inflationary environment. Should any shareholders in some of the more recent issues suffer total capital loss due to overstretched capital structures, the whole investment trust industry could be affected by the consequent adverse publicity. There is a clear need for more sophisticated risk measures relating to the fundamental risks that investors in split securities face.

2.3 CONVENTIONAL TRUSTS

Conventional investment trusts are those without a split share capital structure.[8] They generally invest almost entirely in equities, often with a heavy overseas involvement. The trusts may be categorised in various ways but a useful starting point is to place them in broad geographical categories. The NatWest Securities (NWS) (subsequently BT Alex.Brown) Investment Trust Indices[9] have five main sub-sectors: International; UK; European; Geographical specialists; and Venture & development capital (see Appendix 2A). Table 2.1 shows the number of trusts in the FT-SE Actuaries Investment Trust Index and their aggregate market capitalisation for

Table 2.1: Size of the Conventional Trust Sub-sectors on 31 December 1997

Sub-sector	Number of trusts[10]	Market cap. (£m)
International	32	14,184
UK	40	8,152
European	9	2,106
Geographical	38	5,855
Venture & dev. cap.	7	4,625

Source: BT Alex.Brown

[8] Lloyds corporate capital vehicles and the new housing investment trusts, which can be regarded as separate types of investment, are not generally regarded as conventional trusts.
[9] NatWest Securities, 1995.
[10] Constituents of the FT-Actuaries Investment Trust Index

each of the five sub-sectors on 31 December 1997. The trusts may also issue other securities such as convertible loan stock or warrants.

2.3.1 International trusts

These trusts hold less than 80% of their assets in any one geographical region, and account for around a quarter of the market capitalisation of all conventional investment trusts. Many of the largest investment trusts are international trusts. They combine investment flexibility with the opportunity to diversify by spreading investments over several markets and sectors. The high level of diversification reduces the volatility of the underlying portfolio of assets although the average exposure of the international generalists to the UK is a little over 50%. Management fees are generally low and they are often held as long-term investments so that share turnover by investors is relatively low. For many international trusts, there is a Scottish connection which dates back over a century.

2.3.2 UK trusts

These trusts hold at least 80% of their assets in UK-registered companies, so diversification benefits are restricted to some extent, and there is little foreign currency exposure. They generally offer higher dividend yields than other investment trusts. Venture & development trusts, which invest mainly in the UK, are discussed separately in 2.3.5.

2.3.3 European trusts

These trusts hold at least 80% of their assets in Continental Europe. The sub-sector was boosted in 1994 with the launch of two massive trusts which attracted considerable private investor interest, Kleinwort European Privatisation (KEPIT) and Mercury European Privatisation (MEPIT). Unfortunately, they were launched right at the top of the market and European governments turned out to be slower than Britain to sell off state assets. Both trusts ran into difficulties and slid to large discounts. KEPIT was wound up and MEPIT broadened its mandate.

2.3.4 Geographical specialists

Since the removal of exchange controls in 1979, many geographical specialist trusts have been formed. These provide a vehicle for investment managers, even those managing UK equity funds, with a strong view of a particular geographical region to purchase a diversified portfolio with the desired exposure almost instantly. Other benefits compared with direct investment include economies of scale, lower dealing costs and the provision of specialist investment management. But geographical specialists tend to be volatile. Historically most geographical specialists have concentrated on investment in North America or the Far East, but in recent years, there has been an emphasis on the formation of geographical specialists which invest solely in shares quoted on specific foreign stock exchange(s) located in one particular country ('single-country funds' or just 'country funds'). Some developing countries, such as Taiwan and Korea, restrict access to foreign equity ownership, so that a country fund may be the only readily available vehicle for investing in that particular country.

2.3.5 Venture & development capital trusts

These trusts invest in unlisted companies. They adopt a positive policy of providing capital for buy-outs, start-ups, etc. and a 'hands on' approach to the management of investments. The largest of such trusts is 3i, a constituent of the FTSE 100 Index. Included in this category are the 'new' venture capital trusts, introduced in the 1995 Finance Act, which offer private shareholders certain tax advantages.[11] The purpose of these tax breaks is to encourage start-ups or small companies.

2.3.6 Warrants

Some trusts have warrants in issue. These are similar to long-term call options.[12] They give the holder the right, but not the obligation, to buy ordinary shares in the trust at a predetermined price (the 'exercise price') on one or more future dates. Unlike ordinary shares, warrants are not part of the company's capital. Holders

[11] 'New' venture capital trusts (VCTs) offer 20% up-front income tax relief on investments up to £100,000; investors can defer paying capital gains tax on profits from other investments which are used to buy VCT shares; and there is no income tax on dividends received from the VCT and no capital gains tax on profits from the sale of VCT shares.

[12] Strictly speaking, warrants are not the same as options, because exercising warrants causes dilution.

receive no income and are not entitled to vote except in special circumstances such as liquidation votes.

Most investment trust new issues in recent times have come as packages with 'free' warrants attached, typically one warrant for every five new shares. At a predetermined date, typically two or three months after issue, the warrants may be 'detached' and traded separately.[13] The exercise price of the warrants is usually set at the issue price (roughly equal to NAV) of the trust's ordinary shares. If the share price fails to rise above the exercise price by the expiry date, holders will not subscribe for shares. If, on the other hand, the share price rises to stand above the exercise price at the expiry date, then holders will eventually subscribe. The resulting sale of shares to investors would mean an inflow of funds to the trust, an increase in the issued share capital and dilution[14] of NAV.

When the warrants are traded separately from the shares, the share and warrant packages generally trade at a greater value than issues without warrants. Gemmill and Thomas (1997) argue that this is due to investor 'confusion'. Warrants are not really 'free' as they give investors claims on the fund they already own as shareholders, and this should be reflected in the share price. From the issuer's viewpoint, however, this 'confusion' increases the chance of a successful issue.

Warrants can create a conflict of interest between managers and shareholders. Warrant prices can, at times, drop to a level at which it makes sense for the manager to buy them in to enhance NAV and reduce future dilution. But such action may reduce the future size of the fund (and hence management fees) and also removes a barrier to outside bids.

2.3.7 Convertible loan stocks

A total of fifteen conventional trusts had convertible loan stocks in issue on 31 December 1996 and many of these were only small issues. Holders have the right, at given dates and on specified terms, to convert all or part of their holdings into

[13] In some issues, the warrants are split off and traded separately on the day of issue.

[14] The cash paid for each new share will normally be less than the trust's NAV per share. So NAV per share will normally fall when warrants are exercised.

ordinary shares of the trust. The terms of conversion are normally given in the form of a 'conversion ratio' e.g. 60 ordinary shares per £100 nominal.

2.3.8 Annual management charges

Annual management charges vary from one investment trust to another, ranging from about 0.2% to 2% of assets under management. The very large long-established trusts typically charge fees of around 0.2% to 0.4% whereas more recently launched trusts specialising in narrow or exotic markets (e.g. Latin America) typically charge fees of around 1.5%. The level of management charges influences NAV performance; the higher the management charges, *ceteris paribus*, the worse the expected performance. Some dealing may be necessary so as to adhere to the objectives of the fund, but fund managers may carry out unnecessary dealing with associated costs. The important question is whether this 'unnecessary' dealing leads to improved performance. Proponents of the Efficient Market Theory (see 3.2.1) would not expect that active portfolio strategies would lead to 'excess returns' without the management having access to non-public information.[15]

2.3.9 Net asset value

NAVs are published monthly with many investment trusts nowadays publishing the figures weekly or daily. Published NAVs are generally considered to be reasonably accurate[16] but if a significant proportion of investments held are unquoted, there will be some uncertainty as to the true value of underlying assets. Changes in the values of unquoted investments take time to come through, via changes in the underlying profits, revaluation by the trust's directors and then possibly by a sale or listing of the underlying companies held. There has also been some debate as to what value should be deducted for prior capital. Trust fund managers have argued that investment trusts should be treated as continuing businesses implying that prior capital should be deducted at nominal value. This is consistent with current

[15] In a study of UK pension fund equity portfolios, Adams and Lambert (1998) suggest that there is a 'home and away' effect. Active dealing in overseas equity portfolios is generally detrimental to investment performance whereas for UK equity portfolios there is no significant correlation between activity and returns.

[16] The month end NAVs are generally not known until up to ten days after the month end but Datastream and Reuters estimate NAVs for most investment trusts on a daily basis.

accounting requirements (FRS13).[17] But institutional investors often regard investment trusts as potential takeover candidates, suggesting that NAVs should be calculated with prior capital deducted at market value.[18] The former approach is employed by The Association of Investment Trust Companies and by Datastream in calculating NAVs and is also adopted in this thesis. If there are convertibles or warrants outstanding, it is standard practice in the investment trust industry to make adjustments on a per share basis to give a 'fully diluted' figure i.e. convertibles are assumed to be converted and warrants are treated as exercised if dilution of NAV would occur. This is the approach generally adopted in this thesis.

2.4 DISCOUNT TO NET ASSET VALUE

Investment trusts are characterised by one of the most interesting puzzles in finance - the discount to NAV. There are a number of parts to this puzzle. Trust shares are issued at an average premium to NAV of about 2% (Levis and Thomas, 1995). This premium largely represents underwriting fees and start-up costs which must be subtracted from initial proceeds. Subsequently, often within a matter of months, shares generally trade at a discount. Discounts then fluctuate widely over time and some trusts can on occasions trade at a premium to NAV. At the end of the life of a trust, due to 'open-ending' or liquidation, the discount narrows as the share price rises to meet NAV less liquidation costs.

Investment trust discounts are of interest to market operators, not least because it may be possible to take advantage of discount anomalies to obtain excess returns, assuming market inefficiency. For more than a decade, large 'hedge' funds (arbitrageurs) and 'predatory' funds have been buying trust shares standing on a wide discount and simultaneously selling short a portfolio of securities replicating the assets held by the trust. The arbitrageur's objective is to buy into trusts standing on large discounts to take advantage of pricing anomalies and then encourage the Board to take measures to narrow the discount or realise value in other ways. In practice,

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[17] Under FRS13, companies only have to disclose the market value of their debt in the accounts, they do not have to give market values in their balance sheets.

[18] In a takeover, prior capital is repaid at market value or at the gross redemption yield on an equivalent gilt, plus any redemption penalties.

perfect replication of sale of the underlying portfolio is impossible and the arbitrageur will use a combination of index proxies which are highly correlated with movements in the trust's underyling assets. The explosive growth of derivative security markets has clearly made this type of activity easier and cheaper to carry out. But the more complex, diversified and secretive a trust in the components of its underlying portfolio, the more difficult it is to carry out this arbitrage activity and the greater the required discount anomaly before the arbitrageur will make a move.

Investment trust discounts are also of great interest to investment trust directors and managers because investment trusts standing at a substantial discount to NAV may be vulnerable to takeover. A better understanding of the discount can help trust managers maximise shareholder value and thereby avoid the possibility of unwelcome bids.

A vast array of factors have been advanced by practitioners to explain conventional investment trust discounts but there is considerable argument as to the relative importance of these different factors. Suggested factors giving rise to a discount or premium include: management performance record; management expenses; taxation; marketability; financial gearing; diversification; uncertainty as to true net asset value; specialist portfolios; supply of shares; limited life; dividend yield; liquidation costs; debt should be deducted at fair value; warrants; interest rates; insider ownership; corporate activity and investor sentiment. It is important to note in the context of this thesis that, with the exception of the last two - corporate activity and investor sentiment, each of these factors normally changes only slowly over time.

Investment trust discounts are also of particular interest to academics as they provide an almost unique opportunity to compare the stock market valuation of a company with the value of that company's net assets. One of the most important hypotheses of modern finance theory, market efficiency, can then be tested (see Section 3.2).

2.5 INDUSTRY PERSPECTIVES OF RISK

The methods adopted by investment analysts in assessing the risk of conventional UK investment trusts include: consideration of the investment objectives of the trust; assessment of the management from a risk perspective; financial ratios; and statistical measures of risk. Investors will normally assess the risk profile of a trust in the context of their existing portfolios of assets (and liabilities!).

Whether a trust is a 'specialist' or a 'generalist' is important. Specialisation can take many forms but is principally by geographical region (e.g. Japan, Europe), by sector (e.g. technology, financials) or by style (e.g. income growth, smaller companies). Specialists are normally considered to be higher risk than the more broadly-based general trusts as the latter are cushioned against unexpected events affecting a particular industry or geographical region. Some underlying markets are inherently more volatile (e.g. emerging markets or smaller companies) and, for the geographical specialists, currency movements add to the risk. Concentration of the portfolio in particular companies or sectors within the specialist area restricts the level of diversification and also needs consideration. Furthermore, specialist trusts are subject to fashion so that discounts tend to be more volatile (see Table 5.3).

For general trusts, the degree of international diversification can be assessed by observing the percentage of assets in the different geographical regions. Particular attention would be given to any concentration of the portfolio in risky areas such as emerging markets. Correlations between the returns from shares held in the 'world' market should be less than those between the returns from shares confined to a particular domestic equity market such as that of the UK, so international diversification should reduce risk. International diversification involves foreign currency exposure but changes in exchange rates can offer protection against (unanticipated) higher relative UK inflation as in the long run exchange rates tend to mirror relative inflation in their economies. Foreign currencies may even be regarded as assets in their own right which can be used to reduce the overall portfolio risk. However, currency exposure can be managed independently of the underlying portfolio and this may be carried out with the aim of boosting returns rather than reducing risk.

Fundamental analysts will also assess the management from a risk perspective. Is there a history of the managers taking large 'bets' on individual companies or sectors? Are the managers of the highest integrity? Management can change, so the stability of the trust's management will also be a consideration.

The more important financial ratios used by fundamental analysts in comparing the risk of different investment trusts include gearing, relative discount range, portfolio yield and percentage of total assets which are unquoted.

Investment trusts that issue fixed-income capital acquire the benefits and risks of gearing. The term gearing is normally taken to mean the ratio of total assets to shareholders funds; sometimes fixed-interest and cash investments are deducted from total assets as this effectively reduces the level of gearing. As holders of fixed-income capital are normally entitled to repayment of a fixed amount of capital in a liquidation, any increase or decrease in the value of underlying assets is wholly attributable to the ordinary shareholders. Thus, there is increased variation in NAV with higher levels of gearing. It should be borne in mind, however, that a trust's level of gearing can change, with some trusts actively managing their level of gearing by buying or selling equities for cash.[19]

The magnitude of the discount range over a given period (e.g. the last year) compared with that which is typical for the sector gives an indication of risk associated with changes in discount. Analysts may also judge that a discount which is high (or low) relative to its recent range may be more likely to fall (or rise) with obvious implications for risk in the short term.

A high (or low) portfolio yield is generally associated with low (or high) risk, where portfolio yield is defined as gross revenue expressed as a percentage of gross underlying assets. But the reasons behind the level of the portfolio yield need careful examination. For example, a high yield might reflect the possibility of dividend cuts

[19] Trusts are not active traders in futures contracts, partly because they fear being classed as 'traders' by the Inland Revenue.

from shares in the underlying portfolio or a more concentrated portfolio with some sectors missing. On the other hand, a high portfolio yield might reflect a high level of cash which would be regarded as less risky. Portfolio yield will, of course, be highly correlated with the investment trust shareholders' dividend yield.

Uncertainty as to true net asset value is greatly influenced by the percentage of total assets which are unquoted, despite British Venture Capital Association guidelines being followed by most trusts. Directors' valuations of unquoted investments may be unreliable and historic to some extent, only changing when 'something happens', such as a share stake changing hands.

Statistical measures of risk based on historic returns are increasingly being used in evaluating investment trust risk. The most commonly used and best known statistical measure of risk is standard deviation (or variance) of share returns, often known as 'total risk' or 'volatility'. The measure uses historic returns to estimate the amount by which share returns will deviate from 'mean' return. It incorporates many of the traditional methods of assessing risk in a single statistic, but being estimated from historic data, makes the implicit assumption that returns will behave in the future as they have in the past. Money Management, Micropal and numerous investment banks now publish volatility figures for investment trusts on a regular basis. A paper by Fleming (1995) for the Association of Investment Trust Companies advocated the use of volatility in the risk assessment of investment trusts but the paper was regarded as controversial by many analysts. Although monthly intervals are generally employed, total risk could be calculated using time intervals other than one month. After all, the investor's time horizon will typically be much greater than one month.

For institutions holding diversified equity portfolios, it may seem natural to use the Market Model[20] for risk assessment. Share beta measures the sensitivity of the trust share's return to the return on a chosen market index. Not only share beta but also its two components, NAV beta and discount beta could be calculated.[21] But what

[20] See Appendix 1.

[21] Share return is equal to NAV return plus discount return. It follows that share beta is equal to NAV beta plus discount beta.

should be used for the market index? The FT All Share Index could be used for investment trusts specialising in the UK, but would be inappropriate for other trusts and is seldom used for them in practice.[22] An alternative approach is to use the trust's own benchmark. The extent to which the managers are making large 'bets' on individual companies or sectors could also be measured by the 'tracking error'. This is the annualised standard deviation of the monthly differences between NAV returns and benchmark returns. A low tracking error means that NAV return is usually close to the benchmark return. Calculation of betas and tracking errors against an appropriate benchmark index is carried out by certain investment banks. But there are almost as many benchmarks for investment trusts as there are investment trusts!

Several fund management groups have started issuing risk gradings for their investment trusts. They involve a broad grading of trusts into categories on consideration of their investment objectives, a number of financial ratios and possibly certain statistical measures of risk. These categories might be: well above average risk, above average risk, average risk etc. The risk gradings are for the benefit of private investors.

2.6 SPLIT CAPITAL TRUSTS

The evolution of split capital investment trusts was described in Section 2.2. They may be defined as investment trusts with more than one main class of share capital, offering different rights to income and capital. They are targeted mainly at private investors and are designed to be wound-up by some future date, with most splits having an original term of seven to ten years.[23] If the trust is wound up, its assets are sold and the proceeds are used to pay off the various classes of share capital after meeting the entitlements of holders of debt, if any. Shareholders always have the option to take cash but in practice, the directors and managers often try to retain the

[22] Nevertheless, beta and specific risk are calculated for investment trusts in the London Business School Risk Measurement Service, published quarterly.

[23] They normally have a fixed wind-up date but for some splits there may be a range of wind-up dates and the life of others may be extended if an extension resolution is passed. Recently, there has been a trend towards 'undated splits' in which the ordinary shares are undated. This has been made possible by issuing ZDPs via a subsidiary company. It is the subsidiary which winds up, to be replaced by a follow-on subsidiary if the split structure is to be maintained.

funds under management by encouraging roll-over into an existing trust or restructuring, rather than liquidation.

Investors in splits are more able to take advantage of tax concessions which apply to their individual circumstances. For example, higher rate taxpayers will prefer capital gains, which will be free of tax up to the annual capital gains tax exemption allowance, rather than income. By creating gearing from their financial structures, splits also have the advantage of offering widely different levels of risk to different investors.

The combination of fixed life and gearing means that, unlike with conventional trusts, discount to NAV is not the most important statistic for assessing split capital trust securities. Investors are more concerned with prospective returns, in particular the estimated redemption yield. But despite their obvious attractions for analysis by financial mathematicians, virtually nothing has been written in the academic literature on UK split capital investment trusts.

A number of splits have run into difficulties because the intricate structure reflects the inflationary expectations and the level of nominal interest rates prevailing at the time they were created. Inflationary expectations and nominal interest rates are now generally much lower and, as a result, the whole structure of some splits has been called into question. However, splits have had a resurgence in recent years and are very much 'flavour of the month' at present. This partly reflects the strength of stock markets which encourages investment in risky securities and partly the fact that the discount to NAV of splits (as a package) has held up better than that of conventional trusts.

There are two types of splits - 'traditional splits' and 'quasi-splits' as illustrated in the NWS Split Capital Index[24] (see Appendix 2B).

[24] NatWest Securities, 1995.

2.6.1 Traditional splits

A basic traditional split has its ordinary share capital divided into two distinct categories - income shares and capital shares. Holders of income shares are entitled to all or most of the distributed income and a predetermined capital value (normally par value) on liquidation. Thus, they receive a much higher yield than from the underlying portfolio. Holders of capital shares receive little or no income but are entitled to the remaining assets on liquidation after the income shares have been redeemed.[25] So they obtain geared growth.

A conflict of interest is inevitable in any traditional split because the pursuit of high yield causes lower capital growth. Capital shareholders lack the same management performance monitoring that income shareholders possess through their dividend entitlement, so capital shareholders tend to be prejudiced against. In particular, the practice of dividend stripping and/or holding a larger than intended proportion of fixed-interest securities to meet income objectives will adversely affect capital growth.[26] This conflict of interest can be avoided in quasi-splits (see 2.6.2).

There can be other types of securities in the capital structure of traditional splits, designed to add gearing to both income shares and capital shares, including ZDPs and stepped preference shares. In this case, the capital shares receive what is left (if anything) after all classes of prior capital have been paid their redemption values. Warrants (on the capital shares) may also be issued.

2.6.2 Quasi-splits

Quasi-splits (also known as 'hybrid splits', 'new splits' or 'highly geared splits') always have ZDPs in issue but there is only one class of ordinary share capital,

[25] The Statement of Recommended Practice for the Financial Statements of Investment Trust Companies (AITC, 1995) recommends that 'Investment management fees should be allocated between capital and revenue in accordance with the board's expected long-term split of returns, in the form of capital gains and income respectively, from the entire portfolio of the investment trust company.

[26] Kumur *et al* (1978) address the problem of conflict of interest, between two classes of shareholder, in the selection of investments for dual purpose funds, the US equivalents of UK traditional splits (see 2.7). They develop a Goal Programming model which can be applied to resolve the inherent conflict of interest. Kumur and Philippatos (1979) then provide an empirical demonstration that dual purpose fund managers could have improved their investment selection by the use of Goal Programming methodology. Litzenberger and Sosin (1977) also show that high turnover of dual purpose fund portfolios does not improve investment performance and causes unanticipated redistributions of wealth between income and capital shareholders.

namely income & residual capital shares. When such a trust is wound up, ZDPs are repaid first.[27] This structure avoids the conflict of interest problem described above for traditional splits and is also a sensible structure from a taxation perspective. Again, warrants may also be issued.

2.6.3 The size of the split security markets

Table 2.2 shows the number of issues and aggregate market capitalisation of the different split capital trust securities at 31 May 1999.

Table 2.2: Size of the Split Capital Security Markets on 31 May 1999

Type of security	Number of issues	Market cap. (£m)
Income shares	33	1,027
Capital shares	28	1,040
ZDPs	54	2,432
Stepped preference shares	1	60
Income & residual capital	38	1,784

Source: Lipper Ltd.

Some splits also offer 'units' which are a package of securities, typically one of each type of security held within that trust. This allows arbitrage, thereby encouraging more trading and more efficient pricing.

We now consider in turn the characteristics of the above main categories of shares found in splits.

2.6.4 Income shares

Income shares form part of the capital of a traditional split capital trust. As they receive most if not all of the income generated by the trust's underlying portfolio, they are suitable for investors who require a high income, such as elderly people or non-taxpayers with a potential capital gains tax liability. Some income shares are

[27] In recent years, many quasi-splits have raised significant amounts of bank borrowing.

entitled to a substantial capital repayment when the trust is wound up whereas other income shares are more like annuities, with very little capital repayment.

Given their high yields and the fall in interest rates over recent years, many income shares stand at a price well above the predetermined capital repayment. So investors will suffer a capital loss on redemption and the shares in effect offer investors the choice of receiving returns in a different form.

Income shares can be valued by discounting the estimated future income stream plus capital repayment. Alternatively, the yield to redemption can be calculated, based on the estimated future income stream. If there is doubt as to whether sufficient assets will be available for the capital repayment of the income shares at the wind-up date, a more sophisticated option[28] valuation model is required (see Section 6.2).

2.6.5 Capital shares

Capital shares receive the remaining assets of a traditional split capital trust at the wind-up date, after all other classes of capital have received their entitlement. They receive no income so their return depends entirely on the nominal growth of underlying assets up to the wind-up date. They are the riskiest type of security in a traditional split, apart from warrants, suitable for high rate taxpayers looking for excitement or very long-term investors.

Capital shares of traditional splits often trade at a considerable discount to NAV, but this discount is not comparable to that of a conventional trust because capital shares have no entitlement to income. The discounts of capital shares tend to narrow as equity markets rise and widen as equity markets fall. This is because of a reduction (increase) in value of the income stream paid to income shareholders as a percentage of NAV in rising (falling) markets. But the corresponding income shares are relatively insensitive to changes in the equity markets.

[28] An option is the right, but not the obligation, to buy a specified amount of an underlying asset at a fixed price on a specified date.

Suppose that there is little doubt that there will be sufficient assets at the wind-up date to pay the entitlement of all other classes of capital. Capital shares can then be valued by estimating residual assets on the basis of a predicted nominal growth rate for underlying assets and discounting at a rate of interest which includes an appropriate risk premium. But if there is doubt as to whether sufficient assets will be available at the wind-up date to meet the capital entitlement of all other classes of capital, an option valuation model is required (see Section 6.2).

2.6.6 Zero dividend preference shares

ZDPs pay a fixed capital sum when the trust is wound up before any distribution can be made to ordinary shareholders, income shareholders or capital shareholders. They have no entitlement to income so that, importantly, there is no liability to income tax. Zero coupon bonds (or low coupon bonds) do not offer this tax advantage. The main influence on the prices of ZDPs is movement in the prices of gilts with similar duration.[29] The comparison with gilts has been simplified since the introduction of a gilt strips market in 1997, although tax differences with ZDPs and the appropriate risk premiums for different ZDPs need careful consideration (see Section 6.4).

ZDPs are attractive to investors who need a fixed sum at a future point in time and are able to use their annual exemption allowance to avoid capital gains tax. If sums of money are required at different points in time, an appropriate portfolio of ZDPs can be created. They are suitable for school fees planning and retirement planning.

The tax status of zero dividend preference shares came under threat in the summer of 1995 when an Inland Revenue Consultative Document on the taxation of gilts and bonds was published. Their tax status was confirmed later in the year, largely as a result of lobbying by the investment trust industry itself, but they remain vulnerable to a change in the tax rules given their favourable tax status.

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[29] Duration may be defined as the weighted average of the times of receipts of payments, where the weights are equal to the present value of those payments.

2.6.7 Stepped preference shares

Stepped preference shares pay dividends which rise at a predetermined rate, together with a fixed capital sum when the trust is wound up. They may or may not rank ahead of ZDPs, if any, but rank ahead of all other split securities on wind-up. As with ZDPs, an important criterion in assessing the attraction of stepped preference shares is the redemption yield compared with that of a gilt with similar duration, taking into account taxation and risk.

There were four trusts with stepped preference shares in issue on 31 December 1996.[30] Three (General Consolidated, Fleming Income & Growth and Scottish National) were traditional splits and one (TR Technology) was a quasi-split. Table 2.2 shows that by 31 May 1999, only one stepped preference share, Fleming Income & Growth, remained in issue. We will therefore not consider stepped preference shares further in this thesis.

2.6.8 Income & residual capital shares

These shares, which are also known as 'highly geared ordinary income shares', offer high income plus all the remaining assets of a quasi-split trust at the wind-up date, after the ZDPs (and any other prior capital) have received their capital entitlement. They might be looked upon as equity partly financed through borrowing. The income is relatively low risk but the entitlement to residual capital is relatively high risk. They are suitable for PEP planning and, for some people, in retirement planning.

Valuing these shares using a DCF approach requires an estimate of both income growth and growth of the underlying assets up to the wind-up date. If there is doubt as to whether sufficient assets will be available to repay the ZDPs an option valuation model is required (see Section 6.3).

2.6.9 Industry perspectives of risk for splits

In this section, we define and discuss the main statistics currently used by investment trust analysts in assessing the risk of different types of split securities. Analysts will

[30] Source: NatWest Securities.

not use these statistics in isolation. They will consider other factors such as the quality of management and the quality of the underlying portfolio of assets (normally UK equities). Some splits invest in other splits which can make them risky. Statistical measures such as volatility or beta are rarely used for split securities.

Income shares

Asset cover is the ratio of total assets to the assets required to pay the predetermined redemption amount of the income shares at the redemption date. It gives a rough indication of the risk of the income shareholders not receiving their full entitlement at redemption but does not take the term to redemption into account.

Hurdle rate is the required annual growth rate of total assets to pay the full redemption amount of the income shares. This is a crude measure of the risk for comparative purposes because it does not take into account the yield on the underlying portfolio of assets. Clearly, the higher the yield on the underlying portfolio, the more difficult it is to achieve the required annual growth rate.

Revenue reserve as % of dividend shows the extent to which the current expected or forecast dividend is covered by the existing revenue reserve. This is relevant to the risk assessment of income shares because the dividend is the main element of return.

Capital shares

Hurdle rate is the required annual growth rate of total assets so that the capital shares are repaid at the current share price, after paying off the other shareholders. The higher the hurdle rate, the riskier the capital shares. This does not take into account either gearing or the yield on the underlying portfolio.

Wipe-out rate for capital shares is simply the hurdle rate for income shares. It is the annual growth rate of total assets for which the capital shares just become worthless at redemption.

Gearing may be defined as the ratio of total assets to the assets attributable to capital shareholders. The ratio takes no account of the term to redemption, but it does at least give some indication of short-term price volatility.

A further approach is to calculate the *gross redemption yield* (i.e. the pre-tax internal rate of return to redemption) for different assumed growth rates of total assets. This gives an indication of the sensitivity of the capital shareholders' returns to the growth rate of total assets.

Zero dividend preference shares.

Asset cover and *hurdle rate* are as defined for income shares, and the same critcisms apply as for income shares. Although ZDPs offer a fixed return if repaid in full and held to maturity, they can be volatile in the short term, simply due to changes in the gilt-edged market. This is not reflected in the asset cover or the hurdle rate.

Income & residual capital shares

Hurdle rate, wipe-out rate, gearing[31] and *gross redemption yields* may be used to assess risk, as with capital shares. The same criticisms apply. *Revenue reserve as % of dividend* may also be used in assessing security of income, as with income shares.

2.7 US CLOSED-END FUNDS

The equivalent of investment trusts in the US are known as closed-end funds. They have not been a very popular type of collective investment vehicle, with the market for equity funds no bigger than that of UK investment trusts. Nevertheless, they have attracted much research interest, particularly in respect of the discount anomaly. As a result, most academic studies of UK investment trusts refer to studies of US closed-end funds.

[31] It is debateable as to what should be deducted for the ZDPs in the gearing calculation. The nominal value of the ZDPs is not fixed but moves up over time in line with its predetermined entitlement to capital (i.e. the theoretical amount at which they would be repaid were the trust to be wound up immediately).

US closed-end funds are regulated by the Investment Company Act of 1940 and the securities they issue must be registered with the Securities and Exchange Commission (SEC). The majority of closed-end fund shares are listed on the New York Stock Exchange or the American Stock Exchange, but a small proportion is traded on NASDAQ, the US over-the-counter market (Fredman and Scott, 1991).

The early British investment trusts served as a model for similar vehicles in the US. The first US closed-end fund, the Boston Personal Property Trust, was formed in 1893, twenty-five years after the first UK investment trust (Anderson and Born, 1992). But the first US closed-end fund to employ borrowed funds and thus introduce gearing, Railway and Light Securities Fund, was formed in 1904 (Fredman and Scott, 1991). During the 1920s, the US closed-end fund industry enjoyed phenomenal growth with several hundred funds being formed, often with highly geared structures, to satisfy the speculative appetite of investors at that time. Many of the funds invested in other highly geared funds thereby creating a dangerous pyramid structure of several geared funds. As a result, massive losses were incurred by closed-end fund investors in the 1929 Wall Street Crash (Herzfeld, 1980).

In response to the widespread speculation and abuses of the 1920s, several pieces of legislation were introduced to regulate pooled investment vehicles, including the Securities Act of 1933 and the Securities Exchange Act of 1934. The Investment Company Act of 1940 was comprehensive legislation which regulated the formation, management and public share issues of investment companies. Following the 1940 Act, few closed-end funds were formed in the US for many years. The open-end investment company industry grew to become much more important than that of closed-end funds (Anderson and Born, 1992).

It was not until the 1960s that US individuals again showed any interest in closed-end funds, with many convertible funds and bond funds being issued. Seven dual-purpose funds were also created in 1967, modelled on the British traditional splits. By the end of the 1960s, US closed-end funds were generally trading at a premium to NAV and there were a number of new issues. Seventeen stock funds and fourteen bond funds were issued over the years 1969 to 1973 inclusive. But then most funds started to trade at a discount and new issues effectively stopped for nearly a decade (Fredman and Scott, 1991).

The second half of the 1980s saw dramatic growth in the US closed-end fund industry. In 1985 there were 54 closed-end funds with assets amounting to $8 billion but by 1992 there were over 300 funds worth more than $75 billion. As with UK investment trusts, there was a trend towards specialisation of investment objectives. There were 50 US country funds in December 1994 (Levis and Thomas, 1997) whereas only four such funds existed in December 1984. But a major part of the US closed-end fund industry still consists of bond funds which do not exist in the UK due to the unfavourable tax treatment relative to authorised unit trust bond funds.[32] Most US closed-end funds have unlimited lives and dual-purpose funds have played only a minor role in the US. Indeed, the latter were dependent on a tax loophole which was closed in 1989. As a result, no new dual-purpose funds may be established in the US. All the original dual-purpose funds have now been liquidated or converted to open-end status in accordance with rules laid down in their original charters.

The main differences between US closed-end funds and UK investment trusts concern taxation, ownership structure, gearing and the method of raising new equity capital.

The taxation of US closed-end funds is based on the principle of avoiding double taxation of shareholders, as with UK investment trusts, but there are nevertheless important differences in their tax treatment. A US closed-end fund itself normally has no tax liability. If the fund meets the requirements for investment company status[33], its net income, exclusive of capital gains, is not subject to corporate tax. Dividends received by shareholders are treated as taxable income (Anderson and Born, 1992). But to qualify for exclusion from corporate capital gain tax, closed-end

[32] There are a number of reasons for this including the higher rate of corporation tax that applies to investment trusts, and the special tax regime that applies to authorised unit trust bond funds.
[33] To obtain investment company status, a closed-end fund must distribute at least 90% of its net income (excluding capital gains) as taxable dividends.

funds must distribute net realised capital gains[34] in a given year to shareholders as a capital gains distribution[35] in contrast to UK investment trusts which are required to retain all realised capital gains. Closed-end fund shareholders in turn are liable for their proportion of capital gains tax, regardless of the length of time the shares have been held. So new shareholders inherit a contingent capital gains tax liability. Another difference is that management expenses are not deductible from taxable income for US closed-end funds (Fredman and Scott, 1991).

There is a marked difference in the ownership structure of US closed-end funds and UK investment trusts. Typically, individuals hold a much higher percentage of the equity of US closed-end funds compared with UK investment trusts. Lee *et al* (1991) report that US institutions owned only an average of 6.6% of US closed-end funds in 1988. By contrast, only 27% of UK investment trust shares were held by private individuals in 1997 (HSBC James Capel, 1997).

The level of debt in the capital structure of US closed-end funds is severely restricted by the Investment Company Act of 1940. Funded debt and preferred stock must be covered at least three times and twice by total assets respectively (Anderson and Born, 1992). In contrast, there are no legal restrictions on the level of debt or preference capital for UK investment trusts and some trusts are highly geared.

If an existing US closed-end fund wishes to raise more equity capital, this is generally done by means of a rights issue. In the case of funds trading at a discount, this is against the interests of existing shareholders as they are subscribing cash which immediately falls in value in the hands of the fund. But, unlike in the UK, shareholders are unable to vote against and thereby stop a rights issue. The Investment Company Act of 1940 effectively rules out the possibility of C share issues in the US.

[34] Unrealised capital gains are not taxed and need not be distributed until and unless realised.

[35] Most closed-end funds elect to pay capital gains distributions to shareholders rather than retain them. The corporate capital gain tax rate normally exceeds the corresponding tax rate for individuals. Although the retention of realised gains is rare, any corporate capital gain tax paid is passed on to shareholders on a proportional basis as a tax credit.

2.8 SUMMARY

The investment trust industry has a history dating back to 1868. The early trusts were designed to give smaller investors the opportunity to obtain an interest in a diversified portfolio of securities. During the 1960s and 1970s, however, there was a shift in investment trust ownership away from private investors towards institutional investors. The majority of investment trust shares are currently held by institutions, a situation which the AITC is trying to reverse. The industry has had fluctuating fortunes, with long periods of growth interspersed with shorter periods of contraction.

There has always been an emphasis on overseas investment, but the early trusts invested mainly in fixed-interest securities whereas trusts nowadays invest mainly in equities. Trusts may be categorised according to the geographic region(s) in which they invest, the largest trusts being international generalists investing in a diversified portfolio of equities across different geographical regions and industry sectors. UK generalists and geographical specialists are other important trust categories.

Investment trusts generally trade at a discount to their underlying net assets and there is considerable variation in discounts both across the sector and over time. Many arguments have been advanced by practitioners to explain why trusts stand at a discount but opinions vary greatly on their relative importance. The discount puzzle is of interest not only to market operators and investment trust directors/managers but is also of particular interest to academics largely because of its implications for market efficiency.

The traditional methods adopted by investment analysts in assessing the risk of conventional investment trusts include consideration of the investment objectives of the trust, assessment of the management and financial ratios. But statistical measures of risk based on historic returns are increasingly being used in evaluating investment trust risk. The most commonly used statistical measure is standard deviation of share returns ('volatility').

Split capital investment trusts have more than one main class of share capital, offering different rights to income and capital. Although the first split was not created until 1965, there has been strong growth in the sector in recent years. By creating gearing from their financial structures, splits can offer widely different levels of risk to different investors. Analysts use a variety of ratios and projected rates of return to assess the risk of different split securities. Statistical measures of risk are rarely used.

The equivalent of investment trusts in the US are known as closed-end funds. Although US closed-end funds are similar in many respects to UK investment trusts they have important differences relating to taxation, ownership structure, gearing and the method of raising new equity capital. Some knowledge of US closed-end funds is essential when carrying out academic research on UK investment trusts as there has been much research effort directed at US closed-end funds, particularly in respect of the discount puzzle.

CHAPTER 3 - EFFICIENT MARKETS, ASSET PRICING AND INVESTMENT TRUSTS

3.1 INTRODUCTION

Much research effort has been directed at rational explanations of conventional investment trust (and US closed-end fund) discounts and whether decision rules based on discounts can systematically provide excess returns. This literature is discussed in Section 3.2. An alternative to the efficient market model, the noise trader model, and its relevance to the closed-end fund puzzle, is then discussed in Section 3.3. Section 3.4 reviews work relating to the excess volatility of closed-end funds compared with their underlying net assets. Such excess volatility contradicts the efficient market model but is consistent with the noise trader model.

3.2 MARKET EFFICIENCY AND INVESTMENT TRUSTS

In a perfect market, the market value of an investment trust must equal the net asset value of its underlying portfolio. Otherwise there would be an opportunity for profitable arbitrage. But the introduction of transaction costs, agency costs and other problems breaks the link between market value of the trust and its net asset value. Explanations of the closed-end fund discount puzzle cannot therefore be within the framework of the perfect market model and there is now a vast literature seeking to provide explanations of the discount puzzle which are consistent with the efficient market model.

3.2.1 Efficient Market Hypothesis

Academic research in the 1960s and earlier, mainly on shares quoted on the New York Stock Exchange, seemed to suggest that share prices followed a random walk. That is, successive share prices were independent of each other or, in statistical terms, there was no serial correlation between successive share price changes. Having established randomness, at least to their own satisfaction if not to that of market participants, academics started to search for an explanation as to why there should be randomness. The search led to the concept of market efficiency.

A stock market is said to be *informationally efficient* if it fully reflects all available information (Fama, 1991). In such a market, the activity of investors removes any opportunity for systematically achieving excess returns. There is no way of formally disproving informational efficiency but it is possible to carry out a number of tests.

By the early 1970s, it had become widely accepted among academics that major stock markets, such as New York and London, were close to efficient, in the sense that share prices fully reflected all publicly available information. Most of the tests consisted of event studies. This involved testing the market's reaction to specific announcements (such as takeovers or scrip issues) relating to the shares concerned to see whether, on average, excess returns (after risk adjustment - see 3.2.4) could be obtained by dealing immediately after the announcements.

During the 1980s, however, a number of stock market 'anomalies' were discovered (Dimson, 1988). There seemed to be a considerable degree of return predictability on the basis of certain fundamental variables such as discount to net asset value (not confined to closed-end funds), dividend yields, P/E ratio and market capitalisation. Various seasonal and day-of-the-week patterns were discovered. Furthermore, contrary to the earlier research, it seemed that share returns over time were correlated; they were positively correlated with short return intervals and negatively correlated with longer return intervals.

There are two aspects to an efficient market - informational efficiency and rational fundamental valuation. Informational efficiency means that share prices react instantaneously to embody all new information. But just because new information is assimilated by the market very rapidly, it does not mean necessarily that the market *values* shares rationally. Share price changes in response to new information may build upon an irrational valuation of the shares. The stock market crash of 1987 emphasised the distinction between informational efficiency and rational fundamental valuation. Many commentators (e.g. Arbel *et al*, 1988) argue that, despite the market being close to informational efficiency, a speculative bubble had built up immediately prior to the crash of 1987 and that the crash was the market's way of realigning share prices to a more rational level.

A controversial body of research has developed in the last 20 years to test whether variation in actual prices is consistent with that dictated by variability in fundamentals. This 'excess volatility' literature kicked off with papers by Shiller (1981) and Leroy and Porter (1981) both of which assert that share prices are too volatile to have been generated by a rational valuation process. Excess volatility was typically tested using a variance-bounds test which compared the variability of share returns to the variability of dividends. However, Kleidon (1986) and Marsh and Merton (1986) challenged the statistical validity of the variance-bounds test and the intuitive appeal of this simple test was rather overshadowed by the statistical issues surrounding the actual test procedures used. The controversy has now largely subsided because it became clear that rejection of the constant discount rate model for the present value of dividends is not the same as rejection of the efficient market hypothesis. As Fama (1991) puts it: "It now seems clear that volatility tests are another useful way to show that expected returns vary through time". Even if a variance-bounds test allows variation in expected returns through time, the expected return process may be misspecified.

One aspect of investment trust behaviour which is a powerful test of the efficient market hypothesis is whether or not there is excess volatility of investment trust share returns compared with the volatility of underlying NAV returns. The practical problems associated with comparing dividend volatility with return volatility in most variance-bounds tests are avoided. We might expect the discount rate for the investment trust simply to equal the discount rate for the trust's underlying portfolio.[1] If there is excess volatility of trust share returns then this contradicts the efficient market model.

[1] Elton *et al* (1998) argue that if the closed-end fund sells at a discount, the expected return to investors exceeds the expected return on the underlying portfolio, but this has little impact on the proposed excess volatility test of market efficiency.

3.2.2 Rational explanations of the discount

Much of the research on closed-end funds has concentrated on explaining the discount puzzle and has only indirectly addressed the question of market efficiency. The literature mainly concerns US closed-end funds but some research has been directed at UK investment trusts. Explanations consistent with market efficiency that have been proposed include miscalculation of NAV, agency costs, tax timing and investment opportunities.

Miscalculation of NAV covers a number of theories, the most relevant from the perspective of UK investment trusts being the block discount hypothesis. This states that the current market valuation of underlying assets is calculated using the trading price of a marginal share whereas the proceeds from a liquidation, typically involving the sale of large blocks of shares, would be much lower. Unfortunately, this argument is not consistent with the evidence that large positive abnormal returns are observed when funds are open-ended (Brauer (1984) and Brickley and Schallheim (1985) for the US, and Draper (1989) for the UK). Nevertheless, this block discount factor could still contribute to the existence of discounts.

For US closed-end funds, realised capital gains in the fund must be distributed to shareholders and are then taxed, whereas any capital losses as a result of the distribution cannot be realised by shareholders until their shares are sold. This means that NAVs are generally overestimated. A fund with unrealised capital gains in its portfolio should therefore sell at a discount because new investors will incur a potentially higher capital gains tax liability than would be incurred were they simply to purchase the fund's underlying net assets. However, empirical work on this issue for US closed-end funds has provided mixed results (Malkiel (1977), Brickley *et al* (1991) and Malkiel (1995)). In any case, this tax liability argument does not apply to UK investment trusts because there is no equivalent capital gains tax problem.

A number of investment banks have recently stressed that published NAVs of UK investment trusts are generally overestimated because liabilities are deducted at par value (e.g. Merrill Lynch, 1998). With the reduction in interest rates over recent years, the fair value of investment trust liabilities is generally much higher than par value. Cazenove & Co (1998) estimate that calculating NAVs with liabilities at fair value rather than at par value would reduce the sector average discount by around 2%.

The agency cost theory says that discounts are a consequence of capitalising future management fees or inferior future investment performance. However, for US closed-end funds, Malkiel (1977) finds no correlation between discounts and management fees, and Malkiel (1977), Lee *et al* (1991b) and Pontiff (1995) find no significant relationship between discounts and future NAV performance. On the other hand, Chay and Trzcinka (1999) find that US closed-end stock fund discounts reflect NAV performance over the following year. For UK investment trusts, Draper and Paudyal (1991), using pooled time series / cross-sectional data for the period 1983 to 1986, provide empirical support for the notion that discounts reflect *past* investment performance but this does not mean that discounts reflect *future* investment performance. Draper and Paudyal (1991) do not find statistically significant evidence of management fees affecting UK investment trust discounts. Nevertheless, Prior (1995) still interprets the discount as an agency cost.

Another agency cost explanation for UK investment trusts, proposed by Draper (1989), arises from the fact that UK investment trusts are rarely managed 'in house' but contract out their management to outside specialists. This acts as a disincentive for managers to realise the underlying assets of the trust. Draper provides evidence to suggest that termination (liquidiation or unitisation) costs are very low and that shareholders have lost out due to the agency problem.

Closed-end funds deny taxpayers the valuable tax-timing option available if the fund's underlying shares were held directly. As a result, closed-end funds might be expected to trade at a discount. This is a quite separate argument from that relating to US contingent capital gains tax liabilities discussed above. However, Seyhun and Skinner (1994) find no relationship between the use of tax-motivated trading and US closed-end fund discounts, and many investors (e.g. pension funds) in UK investment trusts do not pay tax, which casts doubt on the importance of this factor.

Investment by closed-end funds in foreign securities may influence discounts because overseas diversification is expensive and time-consuming for investors. Some countries may even impose restrictions on direct foreign ownership, making closed-end funds invested in such countries more valuable and possibly causing a premium. Bonser-Neal *et al* (1990) test for a relationship between announcements of a liberalisation of investment restrictions and changes in US country fund discounts over the period 1981-1989. They report that 80% of the country funds studied experience a significant increase in the discount (or reduction in the premium) around the time of the announcement of a liberalisation of investment restrictions. On the other hand, Malkiel (1977) does not find a significant relationship between discounts on US closed-end funds and a dummy variable which captures whether or not the fund is invested exclusively in foreign securities.

Groups of investors may have different objectives/responses. For example, individual investors may behave differently from institutions. Many commentators, such as Prior (1995), argue that there is a relationship between the proportion of investment trust shares held by institutions and the size of discounts. Institutions can replicate the underlying portfolio without having to pay management fees or incurring the risk of adverse movements in the discount, and are therefore only prepared to buy the investment trust shares at a discount. A related argument is that investment trusts trade at a discount because of poor sales effort and public understanding.

Many arguments have been advanced to explain US and UK discounts but, even taken together, they seem incapable of explaining all parts of the puzzle (see Section 2.4). In particular, they do not explain why there are wide variations in discounts over time. If this discount variation over time is due to market inefficiency, it might be possible to construct profitable decision rules to generate excess returns systematically.

3.2.3 Decision rules

Various studies, both in the US and in the UK, have suggested that it is possible to generate excess returns systematically through decision rules based on discounts.

One of the earliest and best known studies to document an apparently profitable decision rule for US closed-end funds was by Thompson (1978). The study is based on monthly data for 23 closed-end funds over the period 1940-1975. To examine any relationship between discounts and returns, he constructs four separate portfolios of closed-end funds based on the sign and size of their discounts. The funds are rebalanced each year in response to changes in the discounts of constituent funds. The 'all funds' portfolio acts as a control portfolio; at the beginning of each year an equal investment is made in each fund in the sample. The 'premium' portfolio consists of equal investment in all funds trading at a premium at the beginning of each year. The 'discount, equal weights' portfolio consists of equal amounts of funds trading at a discount at the beginning of the year. The 'discount, weighted' portfolio consists of the same funds as the 'discount, equal weights' portfolio but the portfolio weights are proportional to the size of the discount at the beginning of the year.

Thompson shows that investors receive higher risk-adjusted returns from funds trading at a discount compared to other New York Stock Exchange common stocks. Over the 32 year period from January 1940 to December 1971, the 'discount weighted' strategy results in excess returns of over 4% per annum. The 'discount, equal weight' strategy results in an excess return of approximately 2% per annum over the same period. But the 'all funds' strategy does not produce excess returns and the 'premium' strategy results in a negative abnormal risk-adjusted return of approximately 8% per annum. However, Thompson stresses that it is not possible to assess whether the results reflect market inefficiency or the inappropriateness of the Capital Asset Pricing Model which is used to adjust returns for risk (see 3.2.4).

Sias (1998) investigates Thompson's decision rule further by adopting different weights for the constituent funds. He finds that the more weight given to high discount funds, the greater the excess return produced. His most extreme strategy results in an excess return 21% per annum greater than an equally weighted strategy.

Based on weekly data for a sample of 18 closed-end funds over the period 1970-1976, Richards *et al* (1980) suggest that decision rules based on the sale of shares when the discount has narrowed and the purchase of shares when the discount has widened, produces excess returns. This strategy involves setting bounds for the purchase and sales of fund shares according to the level of discount at which they are trading. Anderson (1986) adapts the same strategy as Richards *et al* with a slightly different sample and over the three periods 1965-1969, 1970-1976 and 1977-1984. His findings generally support those of Richards *et al* but both studies ignore transaction costs.

A strategy adopted by Pontiff (1995) also seems to achieve excess returns, even after transaction costs. His approach is to construct seven portfolios of closed-end funds from a total sample of 53 funds, where composition of the portfolio is determined by the size of the discount the previous month. For the period 1965-1985, the average monthly abnormal return for the high discount portfolio and low discount portfolio is 1.75% and -0.55% respectively. Funds with 20% discounts have expected one year returns that are 6% greater than non-discounted funds. The correlation between discounts and future returns is attributed to mean-reversion, not to anticipated future NAV performance.

Corner and Matatko (1979) and Draper and Paudyal (1991) replicate Thompson's decision rule for UK investment trusts. Both studies document higher returns for higher discount trusts compared with lower discount trusts, but the results are not statistically significant in both cases. However, Cheng *et al* (1994) claim to find evidence for excess returns with a simple discount-based decision rule for UK investment trusts. A strategy of buying high discount trusts results in excess returns of about 3.5% per annum whereas a strategy of buying low discount trusts results in negative abnormal returns of about 7% per annum. But whether excess returns could be achieved after transaction costs is debatable.

3.2.4 Models of risk

Almost all tests of the efficient market hypothesis, including decision rules, are conditional upon a particular equilibrium model for returns and are therefore subject to a serious 'joint hypothesis' problem. Thus discovery of an apparent inefficiency could indeed be due to an inefficient capital market but it could also be due to the use of an incorrect equilibrium model for returns, or both. Models of returns that have been employed in empirical work include the Market Model, the Capital Asset Pricing Model (CAPM) and Arbtrage Pricing Theory (APT). It is not apparent that any one of these models is more valid than the others, but results using different models are generally confirmatory.

The most common model for returns in event studies[2] has been the Market Model (see Appendix 1). It segments the return on a share into a market index related component and a return that is residual to the market index as follows:

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$$

where R_{it} is the return on share i in period t.

R_m is the return on the market index in period t.

α_i is the constant return unique to share i.

β_i is a measure of the sensitivity of the return on share i to the return on the market index, and calculated as σ_{im}/σ_m^2

e_{it} is the random residual error in period t, assumed to be independently and normally distributed with zero mean and constant variance.

Estimates of the parameters α_i and β_i are calculated using regression analysis with return data from an estimation period. The 'abnormal' return in the test period is then given by:

$$AR_{it} = R_{it} - (\alpha_i + \beta_i R_{mt})$$

Another model of returns, which has often been employed in testing decision rules (e.g. Thompson (1978) - see 3.2.3), is the CAPM. This also predicts that share returns will be linearly related to a single common factor, the return on the market index. But it is a normative equilibrium model, derived from a set of assumptions which concern investor behaviour and market conditions (see, for example, Adams *et al*, 1993). The CAPM gives a simple relationship between expected return, $E(R_i)$, on share i and its risk, as measured by beta:

$$E(R_i) = R_f + \beta_i \{E(R_m) - R_f\}$$

where R_f is the risk-free rate of return
and $E(R_m)$ is the expected return on the market index.

The above CAPM formula can be transformed from its expectational form into an *ex post* form by assuming that, on average, the expected return on the share is equal to its realised return. The 'abnormal' return in the test period is then given by:

$$AR_{it} = R_{it} - (R_{ft} + \beta_i \{R_{mt} - R_{ft}\})$$

Brown and Warner (1980, 1985) carry out studies using simulated data to examine the performance of models for detecting abnormal returns. They conclude that beyond a simple one-factor market model, there is no evidence that more complex models convey any benefit.

It has become clear in recent years that the *ex post* beta coefficient explains very little, if any, of the difference between actual returns on different shares and portfolios (see, for example, Fama and French, 1992). As a result, multifactor models, in particular that developed by Fama and French (1993), have been increasingly used as models for returns (e.g. Pontiff (1995) - see 3.2.3). A multifactor model is based on the assumption that share returns are linearly related to a number of common factors and can be thought of as a generalisation of the CAPM. If a multifactor model is appropriate for the risk/return trade-off, all expected returns above the riskless rate are due to factor risk premiums.

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[2] Armitage (1995) reviews event study methods and evidence of their performance.

3.3 NOISE TRADING

With the apparent inability to explain the closed-end discount puzzle within the framework of the efficient market hypothesis, recent attempts at explaining the phenomenon have adopted alternative theories that involve investor irrationality. The investor sentiment theory proposed by Lee *et al* (1991a), and developed from the De Long *et al* (1990a) noise trader approach to finance, has been especially popular. We first describe the noise trader model before discussing the investor sentiment theory of closed-end funds and empirical evidence for and against it.

3.3.1 The noise trader model

The 'noise trader' model asserts that rational traders in financial markets interact with noise traders. These noise traders are uninformed market participants who are active traders and stock markets generate irrational valuations because of the their activities. When noise traders are present, large positive returns are immediately followed by further large positive returns in the short term. Their irrational trading activity imparts so much risk to the markets that informed rational operators fear to trade against them. This is because arbitrage by rational investors, who have finite time horizons, is risky and therefore limited.

It is usually assumed that dealing by rational investors will dampen fluctuations caused by noise traders. But this is not necessarily the case if noise traders follow positive feedback trading strategies (De Long *et al*, 1990b). That is, they buy shares when prices rise and sell shares when prices fall. It may pay rational investors to act quickly and buy shares ahead of noise traders. Noise traders then 'jump on the bandwagon' later and push prices even further away from fundamental value. This will further increase volatility.

The existence of noise traders can explain some of the stock market anomalies and empirical results cited in 3.2.1. In particular, if share prices are not determined solely by fundamentals, they may be more volatile than that dictated by fundamentals.

3.3.2 Investor sentiment and the closed-end fund puzzle

Investor sentiment has long been seen as a possible source of discount variation over time. Zweig (1973), for example, considered the differential effect on US closed-end fund shares and their underlying net assets of trading by 'professionals' and 'non-professionals'. He claimed that his approach demonstrated sufficient forecasting value to warrant further investigation. But it was not until the 1990s that the investor sentiment theory became popular and attracted serious scrutiny.

Lee *et al* (1991a) argue that US discount movements are driven by changes in the sentiment of small investors who are the dominant owners of US closed-end funds. Institutional investors fail to offset fully the irrational fluctuating sentiment of small investors because discount movements are cross-sectionally correlated (i.e. systematic) and arbitrageurs have finite time horizons. The discount on closed-end funds is then interpreted as an individual investor sentiment index. The theory *requires* that discounts vary stochastically because it is precisely this discount volatility that is responsible for the underpricing, in equilibrium, of closed-end funds relative to their underlying assets.

Lee *et al* identify a number of testable implications of the theory: discounts across different funds will tend to move together; new issues of closed-end funds will tend to occur when seasoned funds sell at a premium or a small discount; and there should be contemporaneous correlation between closed-end fund discounts and share prices of small firms (which tend to be held by small investors).[3]

3.3.3 Empirical evidence

Lee *et al* (1991a) present empirical evidence based on monthly data for 20 US equity funds over the period July 1965 to December 1985 which supports the investor sentiment theory. Discounts of individual domestic funds tend to move together. The average pairwise correlation of annual changes in discounts among domestic equity funds is 0.389 and the average pairwise correlation of monthly changes in

[3] According to Lee *et al* (1991a), in 1988, the average institutional ownership in the smallest 10% of the firms on the NYSE was 26.5%, the average institutional ownership in the largest 10% of the firms on the NYSE was 52.1% and the average institutional ownership in the US domestic equity closed-end funds was 6.6%.

discounts among domestic equity funds is 0.248. Discounts tend to be low in years when there are many new issues. The mean value-weighted discount[4] at the beginning of years when one or more domestic equity fund started is 6.40% whereas the mean value-weighted discount at the beginning of years when no domestic equity fund started is 13.64%. Finally, closed-end fund discounts tend to narrow when small firm shares perform well and tend to widen when small firm shares perform poorly.

Chen *et al* (1993), using the same data as Lee *et al*, argue that the correctly measured comovement between fund discounts and small firm returns is neither strong enough nor robust enough to support the investor sentiment theory. In particular, they emphasise the virtual disappearance of the relation between discounts and small firm returns in the second half of the test period (1975-1985). In response, Chopra *et al* (1993) defend the evidence presented by Lee *et al* arguing that Chen *et al* select the weakest evidence for criticism. Evidence that changes in discounts across funds are correlated, that new funds start up when seasoned funds are at a premium or a small discount are ignored. But one could argue that these observations can be explained without appealing to investor sentiment theory. If funds are invested in the same type of assets (e.g. US equities) and are held in similar structures, discounts might be expected to move together. And profit maximising behaviour suggests that new funds will be started when seasoned funds are trading at a premium or at a small discount.

Swaminathan (1996) provides additional evidence concerning the relationship between US closed-end fund discounts and small firm returns. If individual investors' optimism pushes stock prices too high then we would expect that as these temporary deviations are corrected, stock prices would tend to fall, implying that future returns will be low. This would give rise to positive covariance between current discounts and future small firm returns. Swaminathan does in fact show that discounts forecast small firm returns, which is consistent with investor sentiment theory. However, Swaminathan also shows that discounts contain information about

[4] Some funds had relatively short life spans and others had missing data points. The value-weighted discount index had monthly memberships ranging from 7 funds to 18 funds.

future economic activity (earnings growth rate and inflation) which is not consistent with investor sentiment theory.

Using the signal extraction technique of French and Roll (1986) to measure noise, Brauer (1993) estimates that on average only about 7% of the variance of weekly changes in US closed-end fund discounts can be attributed to noise trading activity. The sample of funds and the period of observation (1966-1990) are similar to those of Lee *et al.* Brauer concludes that the variability of US closed-end fund discounts over time is far greater than can be accounted for by investor sentiment.

Elton *et al* (1998) find no evidence that small investor sentiment, as measured by the change in discount of US closed-end funds, is an important factor in the return generating process for US equities. Using a five-index model rather than a two-index model reverses the pattern of sensitivities to sentiment across size categories reported by Lee *et al* (1991). Other findings do not support the notion that small investor sentiment is a priced factor.

Ammer (1992) makes the straightforward point that UK investment trusts are owned predominantly by institutional investors, yet the stylized facts about UK investment trust discounts are similar to those for US closed-end funds. Indeed, UK discounts are generally *larger* than US discounts. He argues that the discount puzzle cannot therefore be dismissed as an anomaly concerned with small investors.

Hoskins (1994) shows that US closed-end funds of the same type (stock, bond or international funds) have discounts that move together. It is not clear why this discount comovement occurs as it is not explained by the general sentiment of small investors. If investor sentiment is the explanation, there would have to be three types of sentiment: one for stock funds, one for bond funds and one for international funds.

Hardouvelis *et al* (1993), Suh (1993) and Bodurtha *et al* (1995) study US country funds (i.e. closed-end funds invested in a single country) to test for investor sentiment. Country funds are particularly useful for detecting movements in sentiment because fund prices are determined in the local equity market whereas underlying net asset values are determined in foreign equity markets. Country funds therefore capture the differences between local sentiment and foreign sentiment, unlike the Lee *et al* (1991a) investor sentiment theory which simply measures the differential sentiment between small US investors and those influencing the broader US market. All three studies report a persistent common component in the fluctuations of different country fund discounts, despite exchange rate volatility and the varying degree of investment restrictions imposed by countries. They suggest that this is because fund share prices reflect time-varying sentiments of US investors while their NAVs do not.

Demirgures (1993) examines the structural stability between country fund prices and NAVs using Chow tests. The data consists of weekly fund prices and NAVs for all 40 US country funds trading in April 1993, with observation periods for each fund stretching from the initial offering to April 1993 or date of open-ending, if appropriate. Although NAVs explain much of the variation in country fund prices, the explanatory power is unstable over time, supporting the notion that noise traders who trade stochastically are affecting fund prices.

Levis and Thomas (1997) extend the evidence on the investor sentiment theory to country funds listed in London. Their research supports the notion that movements in the average level of discount for funds invested in a given country reflect the sentiment of UK individual investors, proxied by the volume of retail funds flowing into the corresponding unit trust sector. The same result does not hold for institutional funds flowing into these unit trust sectors. This is surprising because institutions are the dominant investors in UK country funds. However, they find no evidence, at the individual trust level, of a statistically significant relation between variability of the discount and the percentage of equity held by individual shareholders. Levis and Thomas (1997) also observe contemporaneous movement in US and UK discounts of similar country funds suggesting that investor sentiment, but not necessarily small investor sentiment, towards a given country pervades both US and UK stock markets.

3.4 VOLATILITY

This section reviews work relating to the excess volatility of closed-end fund shares. As discussed in 3.2.1, if closed-end fund share returns are more volatile than underlying NAV returns, this contradicts the efficient market hypothesis. Most of the studies concern US closed-end funds, but we would expect the behaviour of UK investment trust discounts to be similar to that of US closed-end funds with the same underlying assets, apart from differences due to ownership structure, gearing or taxation (see Section 2.7).

Section 3.4.1 shows how the variance of closed-end fund share return can be split into three components and 3.4.2 discusses problems confronted in practice when assessing the contribution of these three components. We then look at previous work on variance decomposition for US closed-end funds in 3.4.3. Although there have been no studies dedicated to variance decomposition for US country funds and UK investment trusts, there have been studies for such funds, concentrating mainly on other issues, which provide useful evidence; these studies are discussed in 3.4.4 and 3.4.5 respectively. The literature emphasises the importance of discount volatility as a component of total risk, so 3.4.6 reviews work relating specifically to discount volatility. Finally, 3.4.7 considers the implications of previous research for the empirical work to be undertaken in Chapters 4 and 5. Chapter 4 looks at variance decomposition for UK investment trusts and Chapter 5 looks at the cross-sectional variation in discount volatility for UK investment trusts.

3.4.1 Components of total risk

Variance of share return (total risk) is defined as the average squared difference between the share returns and the mean return. This is an appropriate measure of risk if the distribution of returns (or log returns) approximates a Normal distribution, being symmetric about the mean with few outliers. Variance of return is easy to estimate, simple to understand, and is a measure that is widely used in the financial world.[5]

[5]The downside risk measure analogous to variance is semi-variance. This is the average squared difference between downside returns and the mean return. It is of no benefit, of course, if the distribution is symmetric. The advantage of semi-variance over variance is that it does not increase

Discount variation over time contributes to the variance of returns from investment trust shares. Furthermore, discount changes may be correlated with returns from the underlying net assets of the trust.

We now develop a model which splits the variance of return to shareholders into three components. It is similar to the model derived by Sharpe and Sosin (1974) for US closed-end funds.

Let P_t = share price of investment trust at time t

A_t = net asset value per share at time t

Assume that dividends are added to P_t and to A_t in a consistent way to avoid discount discontinuities. Then 'discount return' for period t is defined as:

$$\log_e(1+R_t^D) = \log_e(P_t / P_{t-1}) - \log_e(A_t / A_{t-1})$$
$$= \log_e(1+R_t^P) - \log_e(1+R_t^A)$$

where $\log_e(1+R_t^P)$ is the share price return in period t

and $\log_e(1+R_t^A)$ is the NAV return in period t.

It is helpful to take logarithms as the returns are additive and their distributions are more symmetric.

Rearranging, we obtain:

$$\log_e\left(1+R_t^P\right) = \log_e\left(1+R_t^A\right) + \log_e\left(1+R_t^D\right)$$

That is, share price return is equal to NAV return plus discount return. The time intervals for these returns could be any length - one week, one month or even a year.

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with the possibility of large upside return, thereby avoiding giving a false impression of the risk involved. But it is rarely used in practice and is not suitable for this study as it cannot conveniently be split into components.

The standard statistical formula for the variance of the sum of two random variables then gives:

$$Var\left\{\log_e\left(1+R_t^P\right)\right\} = Var\left\{\log_e\left(1+R_t^A\right)\right\} + Var\left\{\log_e\left(1+R_t^D\right)\right\}$$
$$+ 2Cov\left\{\log_e\left(1+R_t^A\right), \log_e\left(1+R_t^D\right)\right\}$$

Thus, the variance of share return has been split into the following three components:

a) Variance of NAV return.

b) Variance of discount return[6]

c) Twice the covariance between NAV returns and discount returns.

The volatility of different stock markets and their covariances, which are directly reflected in the variance of NAV return, is a topic that has been well researched (e.g. Solnik, 1996). Less well researched have been the other two components of risk: variance of discount return; and twice the covariance between NAV returns and discount returns.

If investors are rational, the variance of closed-end fund share returns should equal the variance of the corresponding NAV returns, since a closed-end fund share is a claim on the fund's underlying portfolio. For this to be the case, the covariance between NAV return and discount return must be sufficiently negative to cancel out the variance of discount return.

3.4.2 Problems with variance decomposition studies

Short-term volatilities in share prices are partly driven by technical factors related to market imbalances (e.g. liquidity, bid-ask spreads), but the impact of these factors will be diversified away through time. Infrequent trading of shares (hence stale prices) will also bias variance estimates but again this problem will reduce as the return interval is increased, and it will also be less important for larger more marketable closed-end funds.

[6] For monthly returns, the square root of b) is often known as 'discount volatility'.

NAV is derived from the underlying portfolio which contains shares whose prices will have different degrees of staleness. So the NAV time series acts like a moving average of past 'true' prices and may be artificially smooth as a result. This effect will be greater the higher the proportion of unquoted shares in the underlying portfolio. On the other hand, we might expect a sharp jump in the prices of unquoted shares on revaluation. This will act as an outlier and will be squared in calculating the variances.[7] The covariance term could also be affected. But again these effects will have less impact with longer return intervals.

The question of dividend recognition requires consideration (see 4.4.1). When shares go ex-dividend, there is a discontinuity in the share price. For US closed-end funds, the NAV is continually updated for accrued earnings, so dividend recognition of the fund shares and the NAV correspond, and there is no discount discontinuity. But for UK investment trusts, revenue items are excluded from the NAV calculation, so there is a discount discontinuity when the shares go ex-dividend.

All the above problems can be reduced by using longer return intervals, but this means that the period of observation needs to be longer for there to be sufficient data points. Unfortunately, this leads to time series estimation problems. That is, the variances are sample estimates of changing portfolios. As the variances are affected by all the observations in the time series, they will be imperfect estimates of the true variances. This holds for both the variance of the closed-end fund share returns and the variance of NAV returns. Nevertheless, there is an implicit assumption that the estimation problem will affect them identically.

Finally, there is the question of survivorship bias. Shares which do not survive for the entire period of observation are excluded from the data set, and if such shares have particular variance decomposition characteristics, this will bias the results. However, there would seem to be no obvious reason why the variance decomposition of non-survivors should be different from that of survivors.

[7] Empirical evidence in Chapter 5 suggests that this is a less important influence. Correlation between stdevnavr and %u/q is -0.34.

3.4.3 US closed-end funds

There have been two variance decomposition studies for US closed-end funds, one by Sharpe and Sosin (1974) using both annual and quarterly return intervals and one by Pontiff (1997) using monthly return intervals. In addition, there have been a variety of studies, primarily concerned with other matters, which provide evidence concerning the covariance between discount returns and NAV returns.

Variance decomposition studies

Sharpe and Sosin (1974), using annual data from 1933 to 1972 on ten US closed-end funds invested largely in US equities, observe excess volatility of closed-end fund share prices compared with underlying NAV for eight of the ten funds (see Table 3.1, column 5). R^P, R^A and R^D are the returns on the shares, NAV and discount respectively. On average, the standard deviation of return on the shares is approximately 17% greater than that of its underlying net assets. Note also that the correlation coefficient between discount returns and NAV returns is negative for six of the ten funds (column 4). Funds for which the correlation coefficient is significantly different from zero (5% level, two tail test) are indicated by an asterisk.[8] However, an issue which is particularly important for long-term studies of this type is that variances are sample estimates of changing portfolios (see 3.4.2). Note also that it is only a small sample of funds.

Using quarterly returns for eight of the funds over the period 1966 to 1973, Sharpe and Sosin find that the standard deviation of return on a share is approximately 28% greater on average than that of its underlying net assets (Table 3.2). The correlation coefficient in column 4 is negative for five of the eight funds but is not significantly different from zero (5% level, two tail test) for any of the funds.[9] They conclude that a fund's discount and the risk and expected return on the discount play a crucial role in determining the risk and return characteristics of closed-end funds.

[8] Assuming a normal distribution for both NAV returns and discount returns.

[9] Assuming a normal distribution for both NAV returns and discount returns.

Table 3.1: Annual data 1933 - 1972 (% per year)

	(1)	(2)	(3)	(4)	(5)
Fund	$\sigma(R^P)$	$\sigma(R^A)$	$\sigma(R^D)$	$Corr(R^A, R^D)$	$\sigma(R^P)/\sigma(R^A)$
Adams	23.21	20.62	8.87	.09	1.13
Carriers	18.37	18.32	7.07	-.21	1.00
Dominick	17.75	16.00	7.69	-.02	1.11
General Amer.	20.86	18.72	12.77	-.13	1.11
Lehman	18.34	16.95	10.04	-.15	1.08
Madison	27.43	17.60	14.17	.46*	1.56
Niagra	21.42	18.32	11.09	.03	1.17
Surveyor	20.87	15.84	11.85	.12	1.32
Tri Con	21.89	18.14	15.53	-.16	1.21
U.S. For	19.90	19.93	12.00	-.43*	.99
Average					1.17

Table 3.2: Quarterly data 1966 - 1973 (% per quarter)

	(1)	(2)	(3)	(4)	(5)
Fund	$\sigma(R^P)$	$\sigma(R^A)$	$\sigma(R^D)$	$Corr(R^A, R^D)$	$\sigma(R^P)/\sigma(R^A)$
Adams	7.62	7.11	5.70	-.30	1.07
Carriers	7.54	6.75	4.84	-.22	1.12
Dominick	9.04	8.48	4.58	-.19	1.07
Intn. Hold	10.70	7.76	5.26	.33	1.38
Lehman	10.76	8.35	7.03	.00	1.29
Madison	11.41	9.46	8.10	-.11	1.21
Tri Con	8.14	7.83	4.07	-.17	1.04
U.S. For	10.56	8.00	5.04	.27	1.32
Average					1.28

Pontiff (1997), using monthly data from July 1965 to December 1985 on 52 US closed-end funds, including both equity funds and bond funds, calculates the figures ($\%^2$) in Table 3.3.

Table 3.3: Monthly data 1965 - 1985

	(1) $Var(R^P)$	(2) $Var(R^A)$	(3) $Var(R^D)$	(4) $2Cov(R^A, R^D)$	(5) $Corr(R^A, R^D)$
Average	51.15	37.89	37.33	-25.42	-0.34
Median	37.52	24.72	19.62	-7.74	-0.18

Table 3.3 shows negative covariance between discount returns and NAV returns. Pontiff also reports, when bimonthly returns are used, that the negative covariance between discount returns and NAV returns persists.

Pontiff then computes, for each fund in the sample, the natural log of the ratio of the share return variance to the NAV return variance. The average ratio is 0.494 which implies that the variance of the average fund's monthly return is 64% greater than the variance of its underlying NAV return. This means that the standard deviation of the average fund's monthly return is 28% greater than that of its underlying NAV return. This excess volatility is largely idiosyncratic and unrelated to aggregate market risk.

Although Pontiff 's results are biased to the extent that infrequent trading or bid-ask spreads bias the variance estimates, when calculated for two monthly, three monthly and four monthly intervals, the average log variance ratios are still significantly different from zero. And since the magnitude of these biases is the same regardless of the return interval whereas variance increases as the return interval increases, this suggests that the biases are not severe. Log variance ratios for intervals greater than four months are not presented.

Other relevant studies

A number of other studies have looked, directly or indirectly, at the covariance between discount returns and NAV returns, using return intervals ranging from weekly through to annual.

Anderson and Born (1987) regressed weekly share returns for a sample of 17 US closed-end funds invested mainly in domestic equities against corresponding weekly NAV returns over the period 1970 to 1981. For 15 of the 17 funds, the slope coefficient is significantly less than unity at the 5% level implying negative covariance between discount returns and NAV returns.

Using monthly data over the period 1965 to 1985, Chen *et al* (1993) carry out regressions of value-weighted returns of share prices of a portfolio of equity closed-end funds (*RSP*) on value-weighted returns of net asset value of the corresponding closed-end funds (*RNAV*). They obtain the following relationship:

$$RSP = 0.002 + 0.957.RNAV$$

Now since the return on the discount (*RD*) is simply $RSP - RNAV$, we obtain:

$$RD = 0.002 + 0.957.RNAV - RNAV$$

$$\text{Hence} \quad RD = 0.002 - 0.043.RNAV$$

This implies negative covariance between monthly discount returns and monthly *NAV* returns.

Malkiel (1977), using quarterly data over the period 1965 to 1972 for US closed-end funds invested mainly in domestic equities, tests for any systematic relationship between changes in fund discounts ($\Delta DISC$)[10] and changes in the Standard & Poor's 500 Stock Composite Index (*SP*) as part of a time series regression analysis. The regression results are:

[10] The change in discount is a measure of the (absolute) return on the discount.

$$\Delta DISC = -32.67 + 0.20RED + 30.21SP(t) / SP(t-1) + 3.79DUM$$
$$(0.82) \qquad (3.19) \qquad (1.90)$$

where *RED* is redemption less sales of open-end mutual funds

and *DUM* is a dummy variable taking on the value of 1 for observations starting in 1970 and 0 prior to that time. (This is included because in 1970 a major brokerage firm started underwriting and actively selling open-end funds whereas previously it had concentrated on the sale of closed-end funds.)

t-statistics are shown in parenthesis under the coefficients.

Malkiel's results show that fund discounts narrow when the US domestic equity market falls and increases when the market rises. This suggests a negative covariance between discount returns and NAV returns, given that the funds are invested mainly in US domestic equities. Malkiel argues that the negative covariance is due to an increase (or decrease) in the contingent capital gains tax liability (see Section 2.7) as the equity market rises (or falls).

Brickley *et al* (1991) find *positive* and significant covariance between premium changes (or, alternatively, discount returns) and NAV returns, using *annual* data on 37 domestic equity closed-end funds over the period 1954 to 1985. In other words, discounts tend to widen with poor NAV performance (a 'double whammy' effect). This suggests that the negative covariance observed with shorter return intervals may dissipate with annual return intervals.

3.4.4 US country funds

There have been no variance decomposition studies specifically for US country funds, but a number of studies primarily concerned with investor sentiment provide useful evidence on excess volatility and variance decomposition with weekly return intervals.

Hardouvelis *et al* (1993) study 35 US country funds for which at least 9 months of data is available over the period January 1985 to January 1993. They report that discounts vary substantially over time, contributing to a variance in weekly fund returns which is on average three times that of weekly NAV returns. More precisely, the mean of the natural log of the ratio of the share return variance to the NAV return variance is 1.17, with standard error of 0.57. Fund prices are 'sticky' with respect to the host country's stock market. The average local market beta for the NAV return (fund share return) is 0.608 (0.428) with weekly data and 0.718 (0.600) with quarterly data, suggesting negative covariance between discount returns and NAV returns.

Bodurtha *et al* (1995) look at weekly data for 33 country funds trading on US exchanges during the 261-week period covering 1986 to 1990. They report that country fund premiums (and share prices) tend to be more volatile than domestic equity fund premiums (and share prices). There is negative covariance between discount returns and NAV returns but the weekly standard deviation of fund share price changes is still more than twice that of NAV changes.

Klibanoff *et al* (1998) study weekly data for 39 country funds over the period January 1986 to March 1994. The standard deviation of price returns (5.50%) is much greater than the standard deviation of NAV returns (3.49%). They also document systematic underreaction of country fund prices to NAV changes. When NAV rises by one per cent in a given week, fund share prices rise by only 0.64%. Even two weeks after the initial change in NAV, fund share prices only incorporate 80% of the NAV return. This confirms the negative covariance between country fund discount returns and their NAV returns which had been reported in previous research.

3.4.5 UK investment trusts

A few research papers have touched on the question of excess volatility for UK investment trusts, but the main emphasis of each of these papers has concerned other matters. They generally employ monthly return intervals. There is currently no published research that looks directly at the decomposition of the variance of investment trust share returns.

Corner and Matatko (1982) examine the monthly returns on 92 investment trusts over the period 1974 to 1979. The average standard deviation of share price total returns is 8.6% whereas the average standard deviation of net asset value total returns is 5.8%. However, this study covered a period in which discounts varied widely, both cross-sectionally and over time. The sector average discount climbed to around 40% in 1974 and again in 1976 (see Figure 2.1). This period is now viewed in investment trust circles as quite an unusual period.[11]

Armitage and Whittaker (1990) looked at monthly data over the relatively short period from January 1988 to July 1989 for the following investment trust subsectors: UK non-specialists; North America; Europe; Japan; and Far East. They discovered a strong inverse relationship between the size of the sub-sector average discount and the relevant stock market index. Given that NAV returns for specialist trusts will be closely related to movements in the host stock market, this suggests positive covariance between discount returns and NAV returns (a 'double whammy' effect). However, the months studied cover a period of narrowing discounts on UK investment trusts and rising world equity markets.

Draper and Paudyal (1991) regressed monthly changes in the average discount (DD_t) on the 41 largest trusts over the period 1975 to 1986 against the monthly total return on the FT-Actuaries All Share Index (RM_t) together with the average discount change for the previous month. They came up with the following relationship:

$$\begin{array}{l} DD_t = 0.897 \;-\; 0.781RM_t \;-\; 0.198DD_{t-1} \\ \qquad\quad (1.69) \quad (-10.53)^* \quad (-3.26)^* \\ \qquad\qquad\qquad R^2 = 0.41 \end{array}$$

An asterisk indicates significance at the 5% level, with t-statistics shown in parenthesis. There is a clear negative relationship between changes in the average discount and returns on the UK equity market. This implies a positive relationship between discount returns and returns on the UK equity market. As very few of the largest trusts are invested entirely in the UK, this suggests the possible influence of UK market sentiment on trust share prices and hence discounts.

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[11] Due to the capital gains tax rules applying to investment trust shareholders at the time, there was a tax incentive for investors to sell investment trust shares before other shares at times of sharp market falls (e.g. 1974) leading to very wide discounts.

Cheng *et al* (1994) consider monthly discount changes for the 63 investment trusts in the FT-Actuaries All Share Index over the period 1985 to 1989. They show that discounts tend to narrow as the UK equity market rises and widen as it declines. Again, this may be due to UK specific sentiment given that most trusts in the sample are international with NAVs at least partly determined in foreign markets. It could also be interpreted as overreaction in the pricing of trust shares.

Levis and Thomas (1997), using available *daily* data for 43 UK country funds over the period 1985-95, show that when the relevant foreign currency weakens relative to sterling, the discount narrows. And when the local market strengthens, the NAV reflects the change, on average, much more strongly than the market value of the country fund's equity and thus the discount widens. This implies that with *daily* data, there is negative covariance between NAV returns and discount returns. This is consistent with the negative covariance findings for US country funds, with covariance generally calculated using weekly data.

3.4.6 Discount volatility

Both the Sharpe and Sosin (1974) and Pontiff (1997) variance decomposition studies emphasise the importance of discount volatility as a component of total risk. Lee *et al* (1991a) claim that their individual investor sentiment theory explains all parts of the closed-end fund discount puzzle, including discount volatility. Additional evidence from country funds suggests that US investor sentiment causes discount fluctuations for US country funds and UK investor sentiment causes discount fluctuations for UK country funds, with sentiment towards a given country pervading both US and UK stock markets (see 3.3.3). This section reviews other research relating to the underlying reasons for discount volatility and its cross-sectional variation.

Ability to discount arbitrage

Hoskins (1994) concentrates on the analogy between US closed-end fund discount volatility and basis volatility in derivative securities, in that they are both a spread between two highly correlated prices. Basis volatility in derivatives markets is most prevalent when cash-market hedges are difficult to transact, so Hoskins examines all the factors that affect the ability of discount arbitrage traders to buy closed-end fund shares trading at a discount and to sell short the fund's underlying portfolio. He carries out a cross-sectional multiple regression analysis with discount volatility, based on weekly intervals, as the dependent variable. The two most significant explanatory variables are NAV volatility and turnover (number of shares traded as a proportion of shares outstanding) of the fund shares themselves. The former makes it more difficult to hedge the exposure to NAV. The latter is by far the most significant explanatory variable, suggesting that when fund share turnover is high, liquidity is insufficient to keep discounts stable. Thus, low liquidity for fund shares is the primary contributor to discount volatility. This suggests that discount volatility may be related to the size of the closed-end fund. Total assets, which is highly correlated with market value of the closed-end fund and therefore should be a reasonable indicator of the level of liquidity for the fund shares, is indeed negatively correlated with discount volatility. However, it is not a significant explanatory variable in the regression analysis.

Negative autocorrelation of discount movements

If closed-end fund share prices include transient noise, the discount time series will pick up the noise and discount movements will show the same negative autocorrelation as the noise. Negative autocorrelation of discount movements could also be caused by staleness of either the closed-end fund shares or the NAV, or both provided that the staleness is not synchronised.

Hoskins (1994) carries out a time series analysis of weekly discount movements for 28 US domestic equity funds and 24 US international equity (mainly single country) funds over the period 1965 to 1992. The most prominent feature revealed is negative autocorrelation of discount movements. To measure the effect, he computes the discount return autocorrelation for each fund separately and then takes the average. For the domestic equity funds, the average autocorrelations are -0.235 for weekly return intervals and -0.240 for monthly return intervals. For the international funds, the average autocorrelations are -0.228 for weekly return intervals and -0.177 for monthly return intervals. After careful analysis, he attributes the negative autocorrelation effect to a combination of three factors - stale fund share prices, low fund share liquidity and stale NAVs.

Cheng *et al* (1994) consider monthly discount changes for 63 investment trusts over the period 1985 to 1989. They observe high negative autocorrelation in discount changes of -0.234 on average. All but 5 of the 63 trusts have negative autocorrelation coefficients, and almost half are significantly different from zero at the 5% level.

The evidence of negative autocorrelation of discount movements suggests that the importance of discount volatility as a component of total risk will tend to reduce as the return interval is increased.

3.4.7 Implications of published research on volatility for UK investment trusts

In Chapter 4, we investigate the components of total risk for a sample of UK investment trusts. Based largely on the evidence from studies of US closed-end funds, we can expect excess volatility of trust share returns compared with NAV returns. US studies also emphasise the importance of the discount volatility component of total risk but negative autocorrelation, reported for both US discounts and UK discounts, suggests that the importance of discount volatility will reduce as the return interval is increased. While US studies are consistent in reporting negative covariance between discount returns and NAV returns, for return intervals up to quarterly, this may be partly due to the changing value of contingent capital gains tax liabilities as the value of underlying net assets rise and fall. This tax effect does not apply to UK investment trusts and the little evidence available concerning the sign of the covariance term for UK investment trusts is mixed.

In Chapter 5, we analyse the cross-sectional variation in discount volatility for UK investment trusts. Results from Hoskins' study of US closed-end funds suggest that variance of NAV returns and turnover of the trust shares themselves (also known as 'trading velocity') will be significant explanatory variables. Direct evidence from UK country funds suggests that the percentage of a trust's equity held by individual shareholders and the percentage of underlying assets held in the UK should be included as explanatory variables.

3.5 SUMMARY

3.5.1 Market efficiency and investment trusts

Many rational explanations for closed-end fund discounts have been proposed but even taken together, they seem incapable of explaining all parts of the discount puzzle. In particular, none of the arguments explain why discounts fluctuate so widely over time.

There is some support, both in the UK and in the US, for profitable decision rules based on discounts, particularly for investors with low transaction costs, such as institutions. But it is difficult to come to a firm conclusion as regards market inefficiency because of the 'joint hypothesis' problem.

3.5.2 Noise trading

The apparent failure of rational explanations for the discount puzzle casts doubt on the rationality of the market and many researchers have turned their attention to behavioural finance, in particular noise trading, as a possible explanation. In the noise trader model for financial markets put forward by De Long *et al* (1990a, 1990b), rational investors interact in financial markets with noise traders who are less than fully rational. Their model suggests that share prices will be more volatile than that dictated by the fundamentals and that share prices will overreact to changes in the fundamentals. Lee *et al* (1991a) extend the De Long *et al* model. They provide evidence which suggests that discounts are an individual investor sentiment index and that the same investor sentiment risk also affects the returns from small firms and other stocks traded mainly by individual investors. Their theory is compelling because other explanations seem incapable of explaining all parts of the closed-end discount puzzle, but the theory has been the source of much controversy in the last decade. Empirical evidence on the investor sentiment hypothesis for US closed-end funds is mixed. A number of studies on US country funds detect a common component in the fluctuations of their discounts reflecting time-varying sentiments of US investors in general rather than specifically individual investors.

3.5.3 Volatility

Studies of US closed-end funds document fund share returns as being more volatile than their NAV returns but the effect appears to reduce as longer time intervals are employed. Research specifically on US country funds also shows that fund shares are much more volatile than their underlying NAVs. The excess volatility of closed-end fund returns compared with underlying NAV returns in all the published US studies contradicts the efficient market model but is consistent with the noise trader model.

Many studies of US closed-end funds report negative covariance between discount returns and NAV returns. One exception is Brickley *et al* (1991) who find positive covariance using annual data over the period 1954 to 1985. This suggests that the negative covariance relation observed by other authors using return intervals ranging from weekly to quarterly may dissipate when longer return intervals are considered. Negative covariance between discount returns and NAV returns is also observed for US country funds based on weekly data.

The importance of discount volatility as a component of total risk is emphasised for all types of US closed-end fund considered. However, negative autocorrelation in discount changes is also reported, implying that discount volatility is a more important component of total risk for shorter return intervals. The considerable variation in discount volatility across the US closed-end fund sector seems to be related to the ability of discount arbitrage traders to operate successfully, thereby adding liquidity to the market for closed-end fund shares.

Very little research has been carried out on excess volatility for UK investment trusts, although Corner and Matatko report that share prices are much more volatile than NAVs over the somewhat unusual period from 1974 to 1979. Little research has been directed at analysing discount volatility or the covariance between discount returns and NAV returns. We investigate these matters in Chapters 4 and 5.

CHAPTER 4 - COMPONENTS OF TOTAL RISK

4.1 INTRODUCTION

The variance of share returns (total risk) of a closed-end fund can be split into three components. These are: variance of NAV returns; variance of discount returns; and twice the covariance between NAV returns and discount returns (see 3.4.1).

Pontiff (1997) observes a negative covariance term in his study of US closed-end funds, but it is not sufficiently negative to cancel out the variance of discount returns. In other words, US closed-end fund prices are more volatile than fundamentals (excess volatility) even though there is a tendency for prices to underreact to fundamentals (negative covariance). He concludes that, since a share in a closed-end fund is a claim on the fund's underlying portfolio, this excess volatility contradicts the efficient market model.

As explained in Section 2.7, investors in US closed-end funds inherit a contingent capital gains tax liability. Investors will therefore prefer *ceteris paribus* to invest in funds with a lower proportion of unrealised capital gains. Consequently, as the proportion of NAV represented by unrealised capital gains increases so should the discount. Therefore, we would expect, for tax reasons alone, negative covariance between NAV returns and discount returns for US closed-end funds. This issue is not discussed in Pontiff (1997).

UK closed-end funds (investment trusts) do not suffer from the direct capital gains tax effect that applies to US closed-end funds. The underlying fund of an investment trust is exempt from tax on capital gains (see Section 1.2). Investment trusts may therefore be regarded as more suitable vehicles for testing market efficiency.

If movements in investment trust discounts over time reflect investor sentiment and do not conform to the efficient market model, there may be opportunities for investors to generate excess returns systematically by exploiting errors in the pricing of trusts. As discussed in 3.2.3, various studies have claimed that it is possible to make excess returns through decision rules based purely on discounts variation over time. While there is evidence to support these claims historically in the US, the evidence is less convincing for the UK. A proper analysis of variance decomposition for UK investment trusts may allow more sophisticated decision rules to be devised which will be more successful in generating excess returns.

In this chapter, we investigate the importance of the three components of the variance of share returns for UK conventional investment trust shares. The analysis will be carried out using monthly returns over the 15 year period from January 1982 to December 1996, but it will also be carried out using three and six-monthly returns, over three shorter (5 year) periods of observation and separately for three sub-sectors. The results are compared and contrasted where possible with those of the Pontiff (1997) variance decomposition analysis for US closed-end funds.

4.2 VARIANCE DECOMPOSITION, EFFICIENT MARKETS AND NOISE TRADERS

Section 3.2.2 described a number of arguments that have been advanced to explain the discount puzzle within the efficient markets framework but none of them seem capable of explaining the wide variations in discounts over time. The most likely candidate is the agency cost argument in that the capitalised value of management fees will vary as the discount rate varies. However, changes in this discount rate will tend to coincide with changes in the discount rate for the underlying assets, using a discounted dividend approach to valuing the underlying assets. In any case, there is little empirical evidence to support the notion that discounts are a consequence of capitalising future management fees.

In an efficient market, one would expect that the variance of share returns over n time intervals would equal n times the variance of share returns over one time interval. But if noise traders are affecting prices in the market, this would be reflected in mean reversion of the discount. The variance of share returns over n time intervals would then be less than n times the variance of share returns over one time interval, and the relative importance of the variance of discount returns as a component of total risk would be lower with greater return intervals.

Consistent with the De Long *et al* noise trader model is the notion that, in the absence of any capital gains tax complications such as in the US closed-end fund market, discounts will tend to narrow when the value of underlying assets rise. A rise in stock markets will encourage noise traders to look for ways into the market, and investment trusts, which provide a ready-made portfolio of shares, will meet this demand. On the other hand, a fall in the market for the underlying assets will tend to cause a widening of the discount leading to a 'double whammy' effect.

4.3 DATA

The sample consists of the 50 largest trusts as at 31 December 1981 (i.e. those with market capitalisation greater than £10m at that time) which survived until the end of 1996. Concentrating on larger trusts reduces the problem of infrequently traded shares and hence stale share prices distorting variance and covariance estimates. They are all constituents of the FT-SE Actuaries All Share Index on 31 December 1996, with the exception of British Investment Trust.[1] Trusts in the sample are listed in Appendix 3.

The following data were collected from Datastream.

a) monthly share prices, adjusted for capital changes etc
b) monthly undiluted NAVs
c) monthly fully diluted NAVs
d) ex-dividend dates
e) dividend payments

The following data were obtained from the AITC to provide a check for the major results in this chapter. NAV is undiluted. Total return means return including reinvestment of net dividends.

[1] The reason for exclusion of British Investment Trust from the index was that the vast majority of shares were tightly held. The trust was unitised in 1997, after the end of the period of observation.

a) monthly share price total return[2]

b) monthly NAV total return[3]

c) monthly discount to NAV

4.4 MEASUREMENT ISSUES

There are two measurement issues which must be discussed before defining the variables to be used in the analysis, namely dividend recognition and warrants.

4.4.1 Dividend recognition

There is a discontinuity in share prices when they go ex-dividend. For US closed-end funds, the NAV is continually updated for accrued earnings and the underlying NAV reduces by the amount of the dividend on an ex-dividend date[4], so there is no discontinuity in the discount to NAV. But for UK investment trusts, dividend recognition is different for trust shares and the corresponding NAV. Revenue items in the current financial year are excluded in the NAV calculation[5] so that while the share price recognises dividend accrual prior to the ex-dividend date, the corresponding NAV does not. In other words, using terminology borrowed from bond markets, the share price is 'dirty' while the corresponding NAV is 'clean'. As a consequence there is a discount discontinuity at the ex-dividend date for UK investment trusts and hence *ceteris paribus* a corresponding apparent negative discount return. Whether this problem is important in the context of the current investigation is an empirical question which is best answered by examining a sample of trusts.

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[2] Net dividends are reinvested in the shares of the trust company at the time the shares are quoted ex-dividend.

[3] Net dividends paid to shareholders are reinvested in the NAV and assumed to be uniformly distributed throughout the year. Net dividends are actual net dividends paid rather than estimates.

[4] To compute NAV between the ex-dividend date and the payment date for US closed-end funds, dividends are subtracted as an account payable before computing NAV.

[5] This is the case for NAVs published by both Datastream and the AITC. When trusts announce NAVs with accrued earnings included, the earnings are stripped out of the NAV.

Table 4.1 shows the three components of total risk for the first 10 trusts in the main sample[6] over the 5 year period, January 1992 to December 1996[7], using Datastream data.

Table 4.1: Impact of dividend adjustments on components of risk

	Not adjusted			Adjusted			Difference		
Company	Var(navr)	Var(disr)	2xCov	Var(navr)	Var(disr)	2Cov	Var(navr)	Var(disr)	2xCov
ATST	10.2	2.3	1.8	10.2	2.0	1.3	0.0	0.3	0.6
AMTS	14.2	11.8	2.2	14.2	12.1	2.1	0.0	-0.3	0.2
AOT	12.7	2.7	2.2	12.7	2.7	1.9	0.0	0.0	0.4
BNKR	15.2	6.0	-2.7	15.2	6.0	-2.2	0.0	0.0	-0.5
BTI	13.2	4.2	1.1	13.2	4.1	0.8	0.0	0.1	0.3
BSET	12.6	5.6	0.4	12.6	5.6	0.3	0.0	0.0	0.1
BITS	15.6	4.8	0.4	15.6	5.0	-0.3	0.0	-0.2	0.7
BUT	18.4	5.3	-1.0	18.4	5.8	-0.9	0.0	-0.5	-0.1
DIG	19.2	4.6	0.6	19.2	5.0	-1.0	0.0	-0.4	1.5
DNDL	13.5	6.1	-0.9	13.5	5.6	0.1	0.0	0.5	-1.0
Average	**14.5**	**5.3**	**0.4**	**14.5**	**5.4**	**0.2**	**0.0**	**-0.1**	**0.2**

The last two columns show that adjustments[8] to correct for dividend recognition make very little difference to the results.

4.4.2 Warrants

If there is an issue of warrants outstanding, it is normal practice in the investment trust industry to make adjustments to NAV on a per share basis by treating warrants as exercised if dilution of NAV would occur, to give a 'fully diluted' figure. Discounts are then calculated by relating share price to fully diluted NAV. Recently, however, some analysts and researchers (see, for example, Smith New Court (1994) and Gemmill and Thomas (1997)) have argued that once the warrants are traded separately, discounts should be calculated on a 'package' basis. That is, if there are five times as many shares as warrants in issue, share price plus one fifth of the warrant price should be compared with the *undiluted* NAV per share in the discount calculation. This approach fully takes into account the way in which any value for the warrants reduces the value of the shares, but is rarely used in practice. If the approach were to be adopted in our study of components of total risk, for consistency the undiluted NAV should also be used to calculate NAV returns. Again, whether this problem is of any importance in the context of the current investigation is an

[6] In alphabetical order, so effectively randomly selected.

[7] Complete dividend information is not available for the earlier periods from Datastream.

[8] Share prices were 'cleaned' for dividend accrual to eliminate discontinuities at ex-dividend dates.

empirical question which is best answered by examining those trusts which are affected.

Table 4.2 shows the three components of total risk over the 15 year period of observation from January 1982 to December 1996 for all those trusts having warrants outstanding at some point in that period, using Datastream data. Nine trusts are affected out of the total sample of 50 trusts.

Table 4.2: Impact of calculating returns on a 'package' basis

	Fully diluted			Package			Difference		
Company	Var(navr)	Var(disr)	2xCov	Var(navr)	Var(disr)	2xCov	Var(navr)	Var(disr)	2xCov
EDIN	20.8	6.9	6.7	20.8	6.9	7.0	0.0	0.0	-0.3
ENSC	22.1	13.5	3.4	22.9	13.5	3.2	-0.8	0.0	0.1
FLMJ	39.1	23.6	3.4	41.9	24.4	2.1	-2.9	-0.8	1.3
FCP	26.2	20.0	9.7	26.9	21.4	13.2	-0.8	-1.3	-3.5
OIT	25.5	11.5	-4.7	26.1	11.5	-5.0	-0.6	0.0	0.4
SCIN	19.9	7.1	3.2	21.3	7.1	3.7	-1.4	0.0	-0.6
THRG	27.7	25.9	-0.3	27.7	25.9	0.9	0.0	-0.1	-1.2
TRY	24.0	25.2	5.7	24.0	25.2	5.8	0.0	0.0	-0.2
WTAN	25.4	6.3	3.0	25.9	6.5	4.2	-0.6	-0.1	-1.2
Average	**25.6**	**15.6**	**3.3**	**26.4**	**15.8**	**3.9**	**-0.8**	**-0.3**	**-0.6**

The last three columns show that calculating returns on a package basis makes very little difference to the results.

4.4.3 Definition of variables

It is clear from the above that adjustments to account for either dividend recognition or warrants on a package basis, make no qualitative difference to the results. Hereafter we therefore use share prices unadjusted for ex-dividend discontinuities together with fully diluted NAVs, as is normal practice for market participants, using data collected from Datastream. Dividends are then ignored in calculating the three components of total risk.[9]

[9] Dividends are included in the AITC return data, as explained in Section 4.3. This AITC data is used as a check on the main results of this chapter.

4.5 RESULTS

In this section, we examine the components of total risk empirically for UK investment trusts. We are implicitly assuming no structural changes over the period of observation.

4.5.1 Monthly returns from 1/82 to 12/96 (15 years)

Table 4.3a shows the results using monthly data over the entire 15 year period of observation.[10] The variance of share return (total risk) is shown in Column (2). Columns (3), (4) and (5) give the figures for the three components of total risk (all measured in $\%^2$) for each trust in the sample, together with an average figure for the whole sample. Column (6) gives the correlation coefficient between NAV return and discount return.

For the average trust, variance of NAV return represents only about 63% of the variance of share return so there is clear evidence of 'excess volatility'. Variance of discount return represents about 30% of the variance of share return, and twice the covariance is positive and accounts for the remaining 7%.

So with monthly data, variance of discount return makes an important contribution to total risk for the average trust. Note, however, that there is considerable cross-sectional variation in the variance of discount return, ranging from 5.39 to 37.64. In other words, discount volatility is far greater for some trusts than for others. The factors which influence this cross-sectional variation in discount volatility[11] will be analysed in detail in Chapter 5.

Table 4.3b shows the percentage of the total variance represented by its three components for all trusts in the sample. Note that the percentage of total risk represented by the variance of discount return ranges from 17.83% to 65.06%.

[10] Very similar results were obtained using AITC data.

[11] Discount volatility is defined to be the standard deviation, rather than the variance, of monthly discount return.

Table 4.3a: Results with monthly returns from 1/82 to 12/96

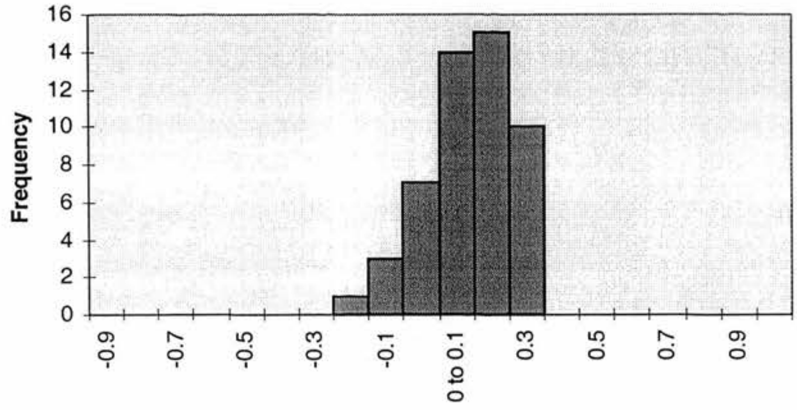
Company	Var(shr)	Var(navr)	Var(disr)	2xCovar	Corr	Fisher
ATST	27.61	17.99	6.51	3.09	0.143	0.144
AMTS	39.02	25.71	12.51	0.79	0.022	0.022
AOT	37.43	22.22	7.69	7.47	0.286	0.294**
BNKR	34.48	23.73	9.52	1.22	0.040	0.041
BTI	30.58	19.44	8.00	3.13	0.126	0.126
BSET	36.30	20.24	10.01	6.01	0.211	0.214**
BITS	22.49	20.38	8.15	-6.01	-0.233	-0.237**
BUT	31.94	23.44	7.69	0.81	0.030	0.030
DIG	35.89	27.86	8.68	-0.65	-0.021	-0.021
DNDL	29.48	21.70	7.81	-0.04	-0.001	-0.001
DWW	30.35	23.26	7.90	-0.80	-0.030	-0.030
EDIN	34.45	20.83	6.92	6.66	0.277	0.285**
ENSC	38.98	22.11	13.50	3.35	0.097	0.097
FAM	50.62	30.78	17.37	2.46	0.053	0.053
FCV	40.01	30.25	9.13	0.62	0.019	0.019
FUT	41.84	24.90	13.37	3.54	0.097	0.097
FFE	73.82	40.50	19.62	13.61	0.241	0.246**
FLMJ	66.15	39.07	23.63	3.44	0.057	0.057
FMN	27.63	16.88	8.12	2.61	0.112	0.112
FOV	42.82	22.51	11.17	9.09	0.286	0.295**
FCP	55.94	26.18	20.05	9.66	0.211	0.214**
FCS	37.72	24.71	12.47	0.53	0.015	0.015
FRCL	36.34	24.49	7.88	3.96	0.143	0.143
GOR	73.34	45.18	15.10	12.98	0.249	0.254**
GVS	49.97	38.43	11.73	-0.19	-0.004	-0.004
GTJA	72.42	39.75	37.64	-4.95	-0.064	-0.064
ELGN	41.04	27.03	9.52	4.46	0.139	0.140
KLC	31.70	18.62	9.61	3.45	0.129	0.129
KOS	35.46	21.67	9.45	4.32	0.151	0.152*
MRCH	36.89	23.60	8.38	4.88	0.173	0.175*
MKI	34.53	25.42	9.03	0.09	0.003	0.003
MNKS	31.34	23.98	6.49	0.87	0.035	0.035
MUT	30.07	21.10	10.24	-1.27	-0.043	-0.043
MYI	31.30	19.94	10.25	1.11	0.039	0.039
MSM	48.22	33.31	13.12	1.78	0.043	0.043
MVN	26.73	15.21	17.41	-5.86	-0.180	-0.182*
OIT	32.32	25.50	11.50	-4.65	-0.136	-0.137
SCAM	28.45	13.69	9.13	5.60	0.251	0.256**
SCEA	37.37	24.05	7.53	5.75	0.214	0.217**
SCIN	30.17	19.88	7.10	3.18	0.134	0.135
SMT	35.87	25.93	6.39	3.52	0.137	0.138
SAT	25.45	18.44	5.39	1.60	0.080	0.081
STS	31.49	20.38	6.57	4.52	0.195	0.198**
SRW	23.38	19.76	7.80	-4.16	-0.167	-0.169*
TMPL	33.36	20.94	8.27	4.12	0.156	0.158*
THRG	53.21	27.66	25.85	-0.30	-0.006	-0.006
TRCD	36.49	24.16	8.59	3.72	0.129	0.130
TRY	54.94	24.00	25.25	5.67	0.115	0.116
TRU	46.32	23.81	13.50	8.96	0.250	0.255**
WTAN	34.76	25.37	6.34	3.03	0.120	0.120
Average	**38.97**	**24.72**	**11.50**	**2.74**	**0.086**	**0.088****

Table 4.3b: Components (%) of total risk, monthly returns from 1/82 to 12/96

Company	Var(navr)%	Var(disr)%	2xCovar%
ATST	65.20	23.59	11.20
AMTS	65.90	32.07	2.03
AOT	59.44	20.57	19.99
BNKR	68.84	27.63	3.53
BTI	63.58	26.17	10.25
BSET	55.81	27.61	16.58
BITS	90.46	36.20	-26.67
BUT	73.38	24.07	2.55
DIG	77.61	24.19	-1.81
DNDL	73.61	26.51	-0.12
DWW	76.62	26.02	-2.64
EDIN	60.53	20.12	19.35
ENSC	56.75	34.65	8.61
FAM	60.83	34.32	4.85
FCV	75.62	22.82	1.56
FUT	59.54	31.98	8.47
FFE	54.93	26.61	18.46
FLMJ	59.07	35.73	5.20
FMN	61.13	29.41	9.46
FOV	52.63	26.13	21.25
FCP	46.85	35.87	17.28
FCS	65.52	33.07	1.41
FRCL	67.42	21.68	10.90
GOR	61.67	20.61	17.72
GVS	76.90	23.48	-0.38
GTJA	54.87	51.96	-6.83
ELGN	65.92	23.22	10.87
KLC	58.78	30.34	10.88
KOS	61.16	26.66	12.18
MRCH	64.02	22.75	13.23
MKI	73.61	26.14	0.25
MNKS	76.51	20.70	2.79
MUT	70.16	34.06	-4.22
MYI	63.72	32.75	3.53
MSM	69.09	27.21	3.70
MVN	56.85	65.06	-21.91
OIT	78.84	35.54	-14.38
SCAM	48.15	32.14	19.71
SCEA	64.41	20.17	15.41
SCIN	65.91	23.55	10.54
SMT	72.34	17.83	9.83
SAT	72.49	21.21	6.30
STS	64.76	20.87	14.37
SRW	84.44	33.31	-17.75
TMPL	62.82	24.83	12.35
THRG	51.98	48.59	-0.56
TRCD	66.24	23.56	10.20
TRY	43.70	45.98	10.32
TRU	51.46	29.17	19.36
WTAN	73.02	18.25	8.73

Figure 4.1 shows the distribution of correlation coefficients between NAV returns and discount returns. The correlation coefficient is positive for 39 of the 50 trusts.

Figure 4.1 - Distribution of correlation coefficients with monthly returns from 1/82 to 12/96



To test whether the correlation coefficient for trust i is significantly different from zero, we calculate the Fisher statistic (Column (7) of Table 4.3a). This is defined as:

$$\hat{z}_i = \frac{1}{2}\log_e \frac{1+\hat{\rho}_i}{1-\hat{\rho}_i}$$

where $\hat{\rho}_i$ is the correlation coefficient.

Then, assuming a normal distribution[12] for both NAV return and discount return, the correlation coefficient for trust i is significantly different from zero at the 5% level (two tail test) if:

$$|\hat{z}_i| > \frac{1.960}{\sqrt{n-3}}$$ where n is the number of observations.

$$= 0.1473$$

Similarly, the correlation coefficient for trust i is significantly different from zero at the 1% level (two tail test) if:

[12] The tests are robust to violations of the normality assumptions and the results are sufficiently strong to remain unchallenged.

$$|\hat{z}_i| > \frac{2.576}{\sqrt{n-3}} \quad \text{where } n \text{ is the number of observations.}$$

$$= 0.1936$$

Trusts for which the correlation coefficient is significantly different from zero at the 5% (1%) level are indicated by an asterisk (two asterisks) in Table 4.3a. In all, 14 (11) of the 50 trusts have correlation coefficients which are positive and significantly different from zero at the 5% (1%) level and only 3 (1) trusts have correlation coefficients which are negative and significantly different from zero at the 5% (1%) level.

The only trust with a negative correlation coefficient which is significantly different from zero at the 1% level is British Investment Trust. The vast majority of the shares in this trust were tightly held by the British Coal Pension Scheme throughout the period of observation. As a result there was an inactive market in the shares, which might explain why the shares tended to be slow in responding to changes in NAV.

There is no direct contingent capital gains tax effect tending to cause a negative covariance term for UK investment trust shareholders as was described in Section 4.1 for US closed-end funds. Nevertheless there may be indirect tax influences. For example, shareholders may be reluctant to sell their shares if, in doing so, they crystallise a capital gains tax liability. They are therefore less likely to sell their shares as the market rises so that discounts may tend to narrow. This is consistent with the observed *positive* covariance between NAV returns and discount returns.

If we assume[13] a homogeneous underlying population ($\rho_1 = \rho_2 = \rho_3 = ... = \rho_{50}$) then the average correlation coefficient is significantly different from zero at the 5% level (two tail test) if:

$$|\bar{z}| > \frac{1.960}{\sqrt{(n-3)}\sqrt{N}} \quad \text{where } N \text{ is the number of trusts}$$

$$= 0.021$$

[13] We also assume independent observations, which is not strictly true because the observations are made over the same period of time, and that the z_i are roughly normally distributed.

The average correlation coefficient is significantly different from zero at the 1% level (two tail test) if:

$$|\bar{z}| > \frac{2.576}{\sqrt{(n-3)}\sqrt{N}}$$

$$= 0.027$$

The calculated value for $\bar{z}$ in Table 4.3a is 0.088 which is positive and significantly different from zero at the 1% level, suggesting a 'double whammy' effect for the investment trust sector. That is, discounts widen when NAVs fall and discounts narrow when NAVs rise. It could be argued, however, that the homogeneous population assumption is invalid because the trusts in the sample are drawn from different sub-sectors, within which there may be different mechanisms working. We will therefore repeat the above test for more homogeneous sub-sectors in 4.5.7.

4.5.2 Comparison with Pontiff's results for US closed-end funds

Pontiff (1997) measures excess volatility by calculating the log variance ratio for each fund in his sample (see 3.4.3). This ratio is defined as the logarithm[14] of the ratio of the variance of share return to the variance of NAV return. It will have a value of zero if variance of share return is equal to the variance of NAV return. The average of the log variance ratios using figures in Table 4.3a is 0.444 which compares with an average of 0.494 using monthly returns for the sample of US closed-end funds in Pontiff's study. This suggests that there is similar excess volatility for UK investment trusts as for US closed-end funds. However, it must be remembered that the period of observation for this UK study is January 1982 to December 1996 whereas the period of observation for Pontiff's study of US closed-end funds is July 1965 to December 1985. Pontiff's sample also includes bond funds as well as equity funds.

Table 4.4 shows the figures for the variance of share returns and the components of total risk for the average trust (from the bottom of Table 4.3a) together with corresponding figures from the Pontiff variance decomposition analysis.

[14] The logarithm of the ratio is taken to reduce skewness.

Table 4.4: Comparison of results for average trust with those of Pontiff (1997)

	Var(shr)	Var(navr)	Var(disr)	2xCovar
UK investment trusts (82-96)	38.97	24.72	11.50	2.74
US closed-end funds (65-85)	51.15	37.89	37.33	-25.42

Note that variance of discount return is far greater on average for US closed-end funds than for UK investment trusts. This is consistent with the apparently greater success of decision rules based on discounts for US closed-end funds (see 3.2.3). But it should again be stressed that the period of observation for the Pontiff (1997) study is earlier than the present study and includes the 1970s which were characterised by extreme movements in closed-end fund discounts both in the US and in the UK.

Note also the large negative covariance term for the average US closed-end fund compared with a small positive (but significant at the 1% level) covariance term for UK investment trusts. The negative covariance term for US closed-end funds may partly reflect the contingent capital gains tax liability effect.

4.5.3 Longer return intervals

Tables 4.5 and 4.6 show the results for the same analysis as in 4.5.1 but with three-monthly return intervals and six-monthly return intervals respectively. The results for longer return intervals emphasise the importance of the investor's time horizon in the risk assessment of investment trusts. There is still evidence of 'excess volatility' but the effect reduces with longer return intervals. For the average trust, variance of NAV return represents 69% of variance of share return for three-monthly intervals and 81% for six-monthly intervals.

Table 4.5: Results with three-monthly returns from 1/82 to 12/96

Company	Var(shr)	Var(navr)	Var(disr)	2xCovar	Corr	Fisher
ATST	80.54	57.38	10.60	12.35	0.250	0.256
AMTS	134.23	93.61	26.39	14.00	0.141	0.142
AOT	125.71	80.15	14.97	30.08	0.434	0.465**
BNKR	118.96	77.11	23.24	18.30	0.216	0.220
BTI	93.88	67.58	12.18	13.89	0.242	0.247
BSET	99.36	64.70	16.54	17.82	0.272	0.279*
BITS	79.65	64.44	15.39	-0.18	-0.003	-0.003
BUT	102.37	77.98	13.52	10.69	0.165	0.166
DIG	102.30	86.21	12.56	3.46	0.053	0.053
DNDL	122.48	91.60	17.21	13.44	0.169	0.171
DWW	116.91	88.96	13.96	13.76	0.195	0.198
EDIN	99.48	73.66	10.84	14.73	0.261	0.267*
ENSC	108.66	77.79	19.78	10.91	0.139	0.140
FAM	177.40	97.29	39.56	39.87	0.321	0.333*
FCV	114.65	96.75	10.82	6.96	0.108	0.108
FUT	139.93	79.46	27.18	32.74	0.352	0.368**
FFE	283.22	162.88	47.72	71.41	0.405	0.430**
FLMJ	225.02	154.88	46.14	23.60	0.140	0.140
FMN	87.27	63.77	18.03	5.39	0.079	0.080
FOV	137.54	82.77	19.45	34.73	0.433	0.463**
FCP	182.49	99.82	38.66	43.28	0.348	0.364**
FCS	135.24	92.91	20.43	21.53	0.247	0.252
FRCL	120.49	85.70	16.32	18.17	0.243	0.248
GOR	279.21	180.92	37.34	59.93	0.365	0.382**
GVS	201.94	169.23	17.75	14.72	0.134	0.135
GTJA	220.74	146.38	68.05	6.20	0.031	0.031
ELGN	142.11	104.06	15.33	22.35	0.280	0.287*
KLC	90.31	62.46	19.55	8.16	0.117	0.117
KOS	131.50	80.69	20.78	29.52	0.360	0.377**
MRCH	101.65	73.58	15.33	12.53	0.187	0.189
MKI	100.15	83.23	17.02	-0.10	-0.001	-0.001
MNKS	121.12	86.85	14.90	19.05	0.265	0.271*
MUT	88.22	67.67	14.57	5.88	0.094	0.094
MYI	96.93	68.21	19.85	8.72	0.118	0.119
MSM	209.20	140.13	24.23	44.09	0.378	0.398**
MVN	90.39	50.75	30.07	9.40	0.120	0.121
OIT	145.08	92.30	19.44	32.78	0.387	0.408**
SCAM	73.01	49.44	16.18	7.27	0.128	0.129
SCEA	98.12	80.81	10.40	6.80	0.117	0.118
SCIN	88.27	67.95	13.86	6.35	0.103	0.104
SMT	126.88	90.59	11.75	24.13	0.370	0.388**
SAT	80.68	59.37	11.21	9.94	0.193	0.195
STS	86.60	64.08	16.35	6.07	0.094	0.094
SRW	96.31	71.57	26.47	-1.71	-0.020	-0.020
TMPL	94.29	62.74	14.68	16.59	0.273	0.281*
THRG	131.33	98.11	58.89	-25.25	-0.166	-0.168
TRCD	104.74	71.21	17.10	16.15	0.231	0.236
TRY	189.22	115.03	58.67	15.27	0.093	0.093
TRU	129.60	88.28	22.76	18.26	0.204	0.207
WTAN	115.95	92.19	15.09	8.52	0.114	0.115
Average	**128.43**	**88.70**	**22.38**	**17.05**	**0.196**	**0.202****

Table 4.6: Results with six-monthly returns from 1/82 to 12/96

Company	Var(shr)	Var(navr)	Var(disr)	2xCovar	Corr	Fisher
ATST	151.38	134.54	13.26	3.46	0.041	0.041
AMTS	243.06	220.66	30.34	-7.67	-0.047	-0.047
AOT	205.69	163.01	16.39	25.40	0.246	0.251
BNKR	199.70	148.76	23.49	26.53	0.224	0.228
BTI	189.75	146.36	13.46	28.93	0.326	0.338
BSET	184.49	144.39	22.30	17.21	0.152	0.153
BITS	171.39	138.46	22.76	9.82	0.087	0.088
BUT	218.46	179.25	16.53	21.93	0.201	0.204
DIG	197.09	182.87	24.66	-10.09	-0.075	-0.075
DNDL	221.25	192.19	26.27	2.70	0.019	0.019
DWW	245.67	196.33	15.05	33.14	0.305	0.315
EDIN	187.94	164.55	16.70	6.46	0.062	0.062
ENSC	220.33	173.88	22.97	22.70	0.180	0.182
FAM	335.24	234.10	60.50	39.28	0.165	0.167
FCV	183.09	168.79	18.00	-3.58	-0.032	-0.032
FUT	274.03	175.75	38.21	58.07	0.354	0.370
FFE	472.09	356.00	40.18	73.37	0.307	0.317
FLMJ	477.68	384.64	66.31	25.85	0.081	0.081
FMN	144.40	141.75	17.13	-14.00	-0.142	-0.143
FOV	278.37	191.08	25.26	59.96	0.432	0.462*
FCP	387.49	258.44	49.23	77.15	0.342	0.356
FCS	279.34	194.13	29.70	53.65	0.353	0.369
FRCL	254.13	198.41	25.96	28.77	0.200	0.203
GOR	502.83	408.89	35.08	56.90	0.238	0.242
GVS	307.53	321.95	17.49	-30.85	-0.206	-0.209
GTJA	449.55	344.15	102.82	2.50	0.007	0.007
ELGN	308.54	227.06	20.75	58.71	0.428	0.457*
KLC	163.33	139.97	23.30	0.07	0.001	0.001
KOS	247.46	182.45	24.11	39.54	0.298	0.307
MRCH	185.14	150.55	24.96	9.31	0.076	0.076
MKI	193.34	157.50	23.76	11.68	0.095	0.096
MNKS	231.54	198.33	23.27	9.61	0.071	0.071
MUT	150.38	146.44	25.47	-20.82	-0.170	-0.172
MYI	203.54	192.62	31.39	-19.79	-0.127	-0.128
MSM	378.02	296.54	28.41	51.30	0.279	0.287
MVN	188.05	138.83	38.89	10.00	0.068	0.068
OIT	289.98	204.20	32.11	51.88	0.320	0.332
SCAM	124.31	113.80	19.78	-8.97	-0.095	-0.095
SCEA	177.87	167.97	15.89	-5.79	-0.056	-0.056
SCIN	179.52	149.78	11.74	17.40	0.207	0.210
SMT	238.80	189.96	10.51	37.04	0.414	0.441*
SAT	152.23	138.33	13.75	0.15	0.002	0.002
STS	166.73	153.22	14.43	-0.90	-0.010	-0.010
SRW	149.98	153.37	27.03	-29.41	-0.228	-0.232
TMPL	174.03	134.76	26.20	12.63	0.106	0.107
THRG	220.68	232.91	65.96	-75.59	-0.305	-0.315
TRCD	130.93	134.63	17.34	-20.34	-0.210	-0.214
TRY	428.87	281.52	71.47	73.35	0.259	0.265
TRU	184.11	169.08	28.24	-12.77	-0.092	-0.093
WTAN	218.75	197.98	14.72	5.84	0.054	0.054
Average	**241.36**	**196.30**	**28.47**	**16.03**	**0.104**	**0.108****

Table 4.7 is a summary table for the average trust with monthly, three-monthly and six-monthly return intervals. The components are expressed as a percentage of the variance of share returns.

Table 4.7: Importance of the three components for the average trust

	Var(navr)	Var(disr)	2xCovar
Monthly	63%	30%	7%
Three-monthly	69%	17%	13%
Six-monthly	81%	12%	7%

The contribution of the covariance term to total risk for the average trust is still relatively small at 13% for three-monthly returns and 7% for six-monthly returns. But 46 of the 50 trusts have positive covariance terms with three-monthly returns and 36 of the 50 trusts have positive covariance terms with six-monthly returns. Figures 4.2 and 4.3 show the distribution of correlation coefficients with three-monthly and six-monthly return intervals respectively.

Figure 4.2 - Distribution of correlation coefficients with three-monthly returns from 1/82 to 12/96

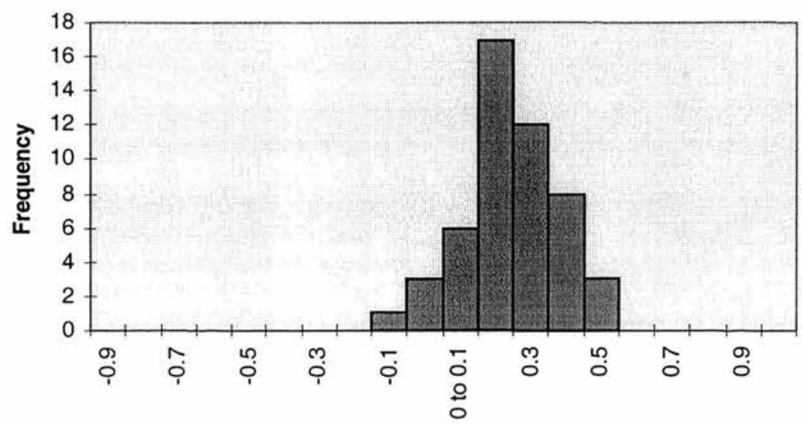
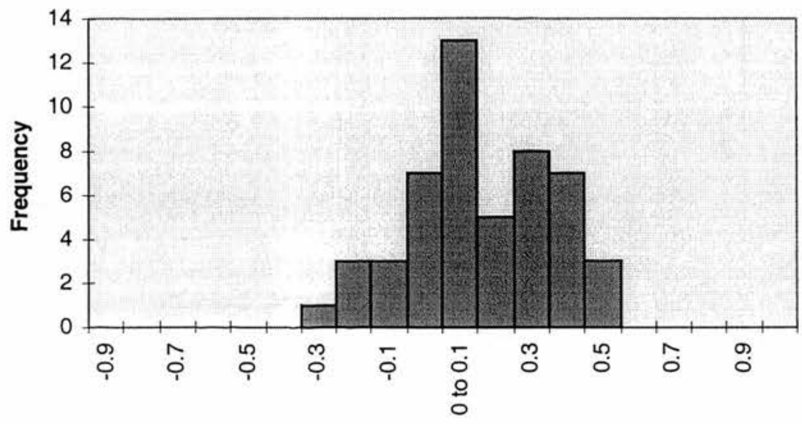


Figure 4.3 - Distribution of correlation coefficients with six-monthly returns from 1/82 to 12/96



Assuming a homogeneous population, the average correlation coefficient is again positive and significantly different from zero at the 1% level (two tail test) for both three-monthly and six-monthly returns (i.e. $\bar{z} > 0.048$ for three-monthly returns and $\bar{z} > 0.070$ for six-monthly returns). The 'double whammy' effect therefore persists for longer time intervals.

For three-monthly returns, variance of discount return contributes 17% of total risk for the average trust compared with 30% for monthly intervals. For six-monthly returns, the contribution of the variance of discount return reduces even further to only 12% of total risk for the average trust. A possible reason for these different results with different return intervals (1 month, 3 months, 6 months) is that trust share prices and NAVs are cointegrated because this would imply negative autocorrelation in discount movements (as reported in previous studies - see 3.4.7).

Figures 4.4, 4.5 and 4.6 show the share price (P_t) and net asset value (A_t) time series for Alliance Trust with monthly, three-monthly and six-monthly data respectively over the period January 1982 to December 1996. The graphs suggest that the two series are in fact cointegrated. A formal cointegration analysis is carried out for the first ten trusts in the main sample in 4.5.4.

Figure 4.4 - Alliance Trust, Monthly observations

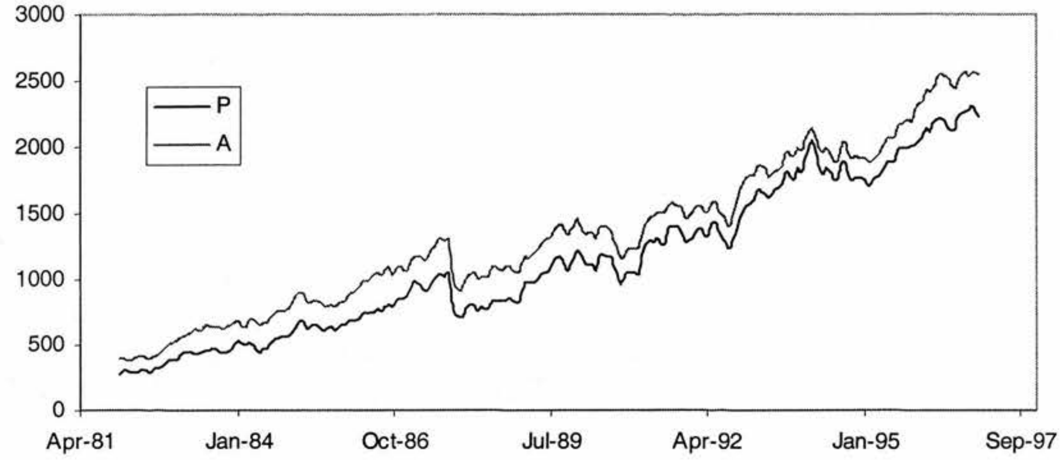


Figure 4.5 - Alliance Trust, Three-monthly observations

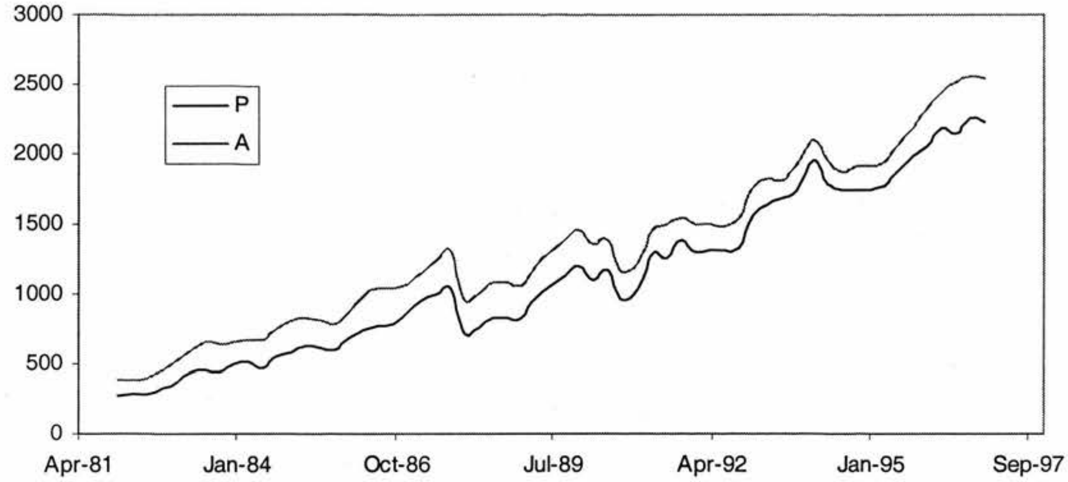
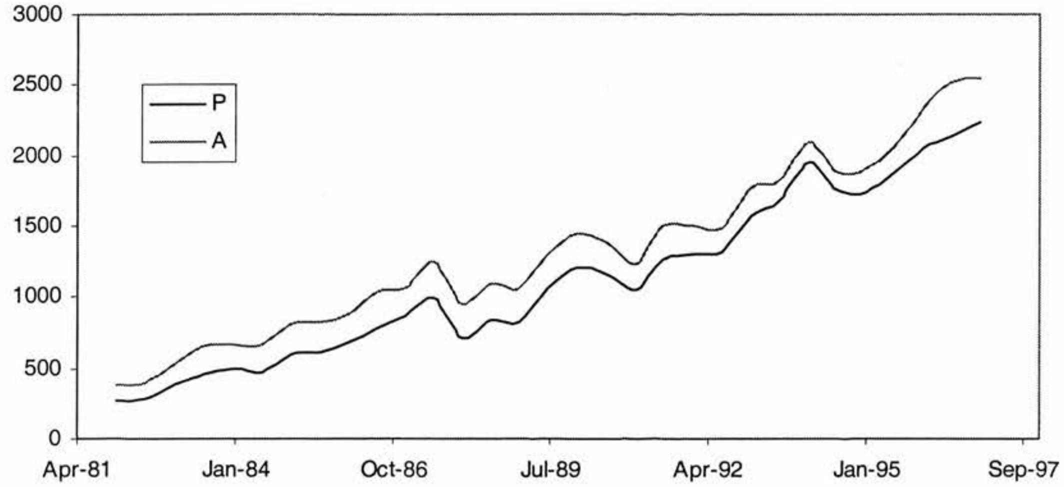


Figure 4.6 - Alliance Trust, Six-monthly observations



4.5.4 Cointegration analysis

If an investment trust's share price (P_t) and net asset value (A_t) are cointegrated (Cheng *et al*, 1994), an error correction model (ECM) may be developed to investigate further the mean reversion of the discount (which is simply A_t minus P_t, expressed as a percentage of A_t). For example, suppose the equilibrium relationship is $P_t = \beta.A_t$. Then the adjustment can be represented as $\lambda(P_{t-1} - \beta.A_{t-1})$ where λ is negative and depends on the frequency of observation of P_t and A_t. The error correction term will become larger the longer the interval between observations because the series converge to the common equilibrium more quickly.

The Engle-Granger approach (Engle and Granger, 1987) is used to test for cointegration for the first ten trusts in the main sample[15] using data for the full 15 year period from January 1982 to December 1996. This approach is described in the literature as a two-step procedure but in fact it involves four steps.

The first step is to carry out a unit root test for the parent series (P_t and A_t). The results are given in Appendix 4A. For monthly, three-monthly and six-monthly returns, using both levels and logs, we fail to reject the null hypothesis of unit root in each case. Therefore, the series are I(1) which is a necessary condition for cointegration.[16]

The next steps are to estimate the cointegrating vector using the equation:

$$P_t = \alpha + \beta A_t + \varepsilon_t$$

and to test for stationarity of the residuals (ε_t). The residuals are in fact stationary (using both levels and logs) for all ten trusts analysed so that, as expected, P_t and A_t are cointegrated. The details are given in Appendix 4B.

The final step is to estimate the equation:

$$\Delta P_t = \alpha + \alpha_1 \Delta A_t + \lambda(P_{t-1} - \beta A_{t-1}) + \eta_t$$

where λ is the speed of adjustment towards the equilibrium.

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[15] In alphabetical order, therefore effectively randomly selected.

[16] The ADF test for the first difference of the series confirms this result.

The value of λ depends on the frequency of the observations as can be seen from Tables 4.8 and 4.9 which give the value of λ (using both levels and logs) for monthly, three-monthly and six-monthly intervals for the ten trusts. The tables show that the estimated value of λ normally increases with less frequent observations, which means that the two series P_t and A_t converge towards the common equilibrium more quickly as the frequency of observation reduces.

Table 4.8: Estimated value of λ (levels)

Trust	Monthly	Three-monthly	Six-monthly
ATST	-0.123	-0.270	-0.262
AMTS	-0.215	-0.399	-0.593
AOT	-0.179	-0.309	-0.532
BNKR	-0.092	-0.167	-0.232
BTI	-0.360	-0.449	-0.399
BSET	-0.072	-0.113	-0.215
BITS	-0.219	-0.317	-0.566
BUT	-0.264	-0.334	-0.421
DIG	-0.095	-0.089	-0.215
DNDL	-0.054	-0.090	-0.160

Table 4.9: Estimated value of λ (logs)

Trust	Monthly	Three-monthly	Six-monthly
ATST	-0.175	-0.310	-0.421
AMTS	-0.250	-0.548	-0.616
AOT	-0.232	-0.398	-0.631
BNKR	-0.187	-0.349	-0.350
BTI	-0.399	-0.588	-0.542
BSET	-0.100	-0.164	-0.257
BITS	-0.226	-0.374	-0.575
BUT	-0.229	-0.368	-0.411
DIG	-0.091	-0.106	-0.211
DNDL	-0.058	-0.107	-0.177

4.5.5 Period of observation split into three five-year sub-periods

Tables 4.10, 4.11 and 4.12 give separate results for three five-year sub-periods - January 1982 to December 1986, January 1987 to December 1991, and January 1992 to December 1996, using monthly returns.[17] Note that the end of the first five-year period is just after 'Big Bang' of the London Stock Exchange (October 1986) and just before the international stock market crash of October 1987. Private investors' consciousness of investment trusts also increased in the second half of the 1980s, which could be reflected in the results for the second and third periods. An alternative approach to the five-year divisions would be to consider bull and bear periods separately, but the entire 15 year period might be described as a bull market. We will, however, consider October 1987 separately.

Comparing Columns (2) and (3) of the three tables shows that the variance of share returns (total risk) and the variance of NAV returns are much higher in the second five-year period reflecting the turbulent equity markets worldwide during and after the October 1987 crash.

Comparing Column (4) of the three tables suggests a downward trend in discount volatility over time. This may reflect some market participants' growing belief in the success of trading strategies based on selling low discount trusts and buying high discount trusts (see 3.2.3). Another possible reason for the reduction in discount volatility could be the reduction in transaction costs. The bid-ask spread has reduced considerably, particularly since Big Bang in October 1986. Also, transfer stamp duty for share purchases was 2% at the beginning of 1982 but was reduced to 1% in March 1984 and reduced further to 0.5% in October 1986.

Comparing the results for the three sub-periods suggests consistency over time for the variance of discount returns. High (or low) variance of discount return for a trust in one period is generally followed by high (or low) variance of discount return for the same trust in the subsequent period. This is interesting as it indicates that the variance of discount return may be predictable. In Chapter 5 we seek to explain the cross-sectional variation in discount volatility.

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[17] Very similar results were obtained using AITC data.

Table 4.10: Results with monthly returns from 1/82 to 12/86

Company	Var(shr)	Var(navr)	Var(disr)	2xCovar	Corr	Fisher
ATST	23.76	13.62	10.88	-0.73	-0.030	-0.030
AMTS	31.57	20.92	12.89	-2.19	-0.067	-0.067
AOT	29.24	15.09	12.82	1.32	0.048	0.048
BNKR	29.05	14.41	15.90	-1.23	-0.041	-0.041
BTI	24.60	11.40	13.09	0.11	0.004	0.004
BSET	32.53	15.30	14.29	2.89	0.098	0.098
BITS	15.42	13.18	8.36	-6.02	-0.287	-0.295*
BUT	23.31	14.88	9.89	-1.44	-0.059	-0.059
DIG	24.13	14.33	13.24	-3.38	-0.123	-0.123
DNDL	16.70	10.59	7.18	-1.06	-0.061	-0.061
DWW	24.39	16.89	11.83	-4.26	-0.151	-0.152
EDIN	27.26	14.42	10.38	2.43	0.099	0.100
ENSC	34.45	17.86	22.49	-5.80	-0.145	-0.146
FAM	42.48	26.97	20.29	-4.70	-0.100	-0.101
FCV	23.99	16.08	8.59	-0.67	-0.029	-0.029
FUT	25.74	19.24	17.16	-10.47	-0.288	-0.297*
FFE	53.55	29.51	24.97	-0.91	-0.017	-0.017
FLMJ	49.39	33.57	21.92	-6.00	-0.111	-0.111
FMN	22.12	15.51	11.93	-5.23	-0.192	-0.195
FOV	38.17	17.96	17.82	2.35	0.066	0.066
FCP	33.80	17.32	22.12	-5.54	-0.142	-0.143
FCS	27.21	18.28	12.69	-3.70	-0.121	-0.122
FRCL	30.94	18.73	12.06	0.15	0.005	0.005
GOR	34.21	19.77	12.40	2.00	0.064	0.064
GVS	28.11	14.90	13.30	-0.09	-0.003	-0.003
GTJA	85.64	47.96	48.87	-11.01	-0.114	-0.114
ELGN	26.80	20.08	12.89	-6.06	-0.188	-0.191
KLC	22.28	13.29	11.58	-2.55	-0.103	-0.103
KOS	29.68	13.32	15.48	0.87	0.030	0.030
MRCH	27.20	12.87	12.45	1.85	0.073	0.073
MKI	15.90	13.69	12.03	-9.66	-0.376	-0.396**
MNKS	23.52	18.15	10.79	-5.34	-0.191	-0.193
MUT	31.56	12.16	16.93	2.43	0.085	0.085
MYI	31.40	13.91	14.63	2.82	0.099	0.099
MSM	38.10	16.87	17.40	3.77	0.110	0.110
MVN	14.49	13.94	11.33	-10.60	-0.422	-0.450**
OIT	26.89	26.49	17.44	-16.75	-0.390	-0.411**
SCAM	23.77	9.38	13.71	0.68	0.030	0.030
SCEA	29.93	15.64	12.11	2.15	0.078	0.078
SCIN	28.06	14.28	13.10	0.67	0.025	0.025
SMT	25.20	18.41	9.81	-2.97	-0.110	-0.111
SAT	20.26	13.94	8.44	-2.09	-0.096	-0.096
STS	24.25	13.31	9.15	1.76	0.080	0.080
SRW	12.80	10.67	5.37	-3.19	-0.211	-0.214
TMPL	27.49	13.64	11.68	2.13	0.085	0.085
THRG	33.55	10.07	16.58	6.79	0.263	0.269*
TRCD	30.91	14.30	13.98	2.58	0.091	0.092
TRY	32.78	8.73	18.45	5.51	0.217	0.221
TRU	34.50	8.58	19.48	6.34	0.245	0.250
WTAN	29.37	17.95	11.02	0.39	0.014	0.014
Average	**29.45**	**16.65**	**14.46**	**-1.63**	**-0.045**	**-0.047***

Table 4.11: Results with monthly returns from 1/87 to 12/91

Company	Var(shr)	Var(navr)	Var(disr)	2xCovar	Corr	Fisher
ATST	44.71	30.07	6.52	7.99	0.285	0.293*
AMTS	57.90	42.41	13.26	2.19	0.046	0.046
AOT	65.62	38.70	7.80	18.80	0.541	0.606**
BNKR	55.90	41.44	6.89	7.45	0.220	0.224
BTI	48.42	33.62	6.91	7.76	0.254	0.260*
BSET	57.90	32.28	10.37	15.00	0.410	0.436**
BITS	31.47	32.49	11.59	-12.41	-0.320	-0.331*
BUT	50.29	37.22	8.16	4.83	0.138	0.139
DIG	59.58	50.19	8.37	1.00	0.024	0.024
DNDL	53.14	40.30	10.12	2.68	0.066	0.066
DWW	47.36	38.37	8.65	0.33	0.009	0.009
EDIN	55.55	34.34	7.14	13.84	0.442	0.475**
ENSC	56.36	36.07	10.53	9.60	0.246	0.251
FAM	82.62	48.06	22.04	12.32	0.189	0.192
FCV	66.21	57.07	9.81	-0.66	-0.014	-0.014
FUT	65.92	39.02	13.57	13.11	0.285	0.293*
FFE	111.44	48.10	26.47	36.25	0.508	0.560**
FLMJ	97.01	44.61	36.65	15.48	0.191	0.194
FMN	34.66	23.07	6.41	5.09	0.209	0.213
FOV	70.05	35.44	12.46	21.78	0.518	0.574**
FCP	94.86	38.05	29.25	27.10	0.406	0.431**
FCS	60.34	42.39	15.14	2.75	0.054	0.054
FRCL	58.66	40.52	8.03	9.95	0.276	0.283*
GOR	143.68	84.08	25.51	33.51	0.362	0.379**
GVS	86.14	74.46	16.87	-5.10	-0.072	-0.072
GTJA	79.35	38.90	38.20	2.21	0.029	0.029
ELGN	76.75	46.81	11.78	17.85	0.380	0.400**
KLC	46.53	29.17	11.02	6.24	0.174	0.176
KOS	57.53	36.65	10.46	10.25	0.262	0.268*
MRCH	56.70	39.92	7.41	9.22	0.268	0.275*
MKI	65.12	46.26	9.54	9.16	0.218	0.222
MNKS	52.80	37.99	5.84	8.83	0.296	0.306*
MUT	40.51	37.95	9.53	-6.85	-0.180	-0.182
MYI	42.98	30.90	10.00	2.05	0.058	0.058
MSM	67.30	55.79	15.58	-4.01	-0.068	-0.068
MVN	40.57	22.90	26.33	-8.52	-0.174	-0.175
OIT	47.03	36.39	11.39	-0.73	-0.018	-0.018
SCAM	47.01	23.20	10.68	12.92	0.410	0.436**
SCEA	59.89	42.14	6.24	11.32	0.349	0.364**
SCIN	46.42	34.80	5.59	5.93	0.212	0.216
SMT	57.86	43.02	5.43	9.25	0.303	0.313*
SAT	38.58	31.18	4.37	2.97	0.127	0.128
STS	49.74	32.61	5.84	11.11	0.403	0.427**
SRW	42.15	35.88	8.59	-2.28	-0.065	-0.065
TMPL	50.61	34.29	7.23	8.93	0.284	0.292*
THRG	66.69	49.99	37.76	-20.70	-0.238	-0.243
TRCD	56.74	42.51	7.20	6.91	0.197	0.200
TRY	73.86	48.99	20.96	3.85	0.060	0.060
TRU	67.06	46.90	10.86	9.14	0.203	0.205
WTAN	57.76	45.11	5.82	6.71	0.207	0.210
Average	**60.87**	**40.65**	**12.84**	**7.25**	**0.179**	**0.188****

Table 4.12: Results with monthly returns from 1/92 to 12/96

Company	Var(shr)	Var(navr)	Var(disr)	2xCovar	Corr	Fisher
ATST	14.36	10.21	2.32	1.80	0.185	0.187
AMTS	28.29	14.23	11.78	2.25	0.087	0.087
AOT	17.69	12.74	2.71	2.21	0.188	0.191
BNKR	18.47	15.23	6.01	-2.73	-0.143	-0.144
BTI	18.46	13.17	4.19	1.09	0.073	0.073
BSET	18.55	12.56	5.59	0.40	0.024	0.024
BITS	20.73	15.59	4.77	0.37	0.021	0.021
BUT	22.66	18.39	5.27	-0.99	-0.050	-0.050
DIG	24.30	19.17	4.56	0.57	0.030	0.030
DNDL	18.72	13.53	6.11	-0.91	-0.050	-0.050
DWW	18.60	13.95	3.44	1.19	0.086	0.086
EDIN	21.13	13.49	3.42	4.15	0.305	0.315*
ENSC	26.39	12.35	7.91	6.02	0.305	0.315*
FAM	27.47	17.84	10.20	-0.56	-0.021	-0.021
FCV	30.33	17.96	9.20	3.11	0.121	0.122
FUT	34.07	16.32	9.75	7.86	0.312	0.322*
FFE	56.18	42.79	8.05	5.24	0.141	0.142
FLMJ	51.12	37.51	12.98	0.62	0.014	0.014
FMN	25.80	11.90	6.22	7.56	0.439	0.471**
FOV	19.76	13.51	3.56	2.65	0.191	0.193
FCP	40.32	23.15	9.42	7.62	0.258	0.264*
FCS	26.30	13.74	9.95	2.58	0.110	0.111
FRCL	19.58	14.06	3.76	1.73	0.119	0.120
GOR	42.83	31.43	7.90	3.44	0.109	0.110
GVS	33.81	23.63	5.39	4.70	0.208	0.211
GTJA	52.44	30.32	26.67	-4.48	-0.079	-0.079
ELGN	19.63	13.79	4.22	1.60	0.105	0.105
KLC	26.40	13.24	6.53	6.52	0.350	0.366**
KOS	17.99	14.38	2.57	1.01	0.083	0.084
MRCH	27.07	18.23	5.40	3.38	0.170	0.172
MKI	22.21	15.99	5.79	0.42	0.022	0.022
MNKS	17.58	15.80	2.99	-1.19	-0.087	-0.087
MUT	17.78	12.87	4.50	0.40	0.027	0.027
MYI	19.14	14.47	6.43	-1.74	-0.090	-0.090
MSM	38.25	26.75	6.70	4.71	0.176	0.178
MVN	23.79	7.70	15.05	1.02	0.047	0.047
OIT	23.12	13.54	6.05	3.48	0.192	0.194
SCAM	14.78	8.50	3.26	2.98	0.283	0.291*
SCEA	22.33	14.24	4.45	3.58	0.225	0.229
SCIN	15.72	10.37	2.78	2.52	0.235	0.239
SMT	24.01	15.77	4.14	4.03	0.250	0.255
SAT	17.41	10.15	3.52	3.68	0.308	0.318*
STS	19.89	14.56	4.74	0.59	0.035	0.035
SRW	15.04	12.33	9.55	-6.73	-0.310	-0.321*
TMPL	22.22	14.59	6.11	1.49	0.079	0.079
THRG	58.38	21.50	23.91	12.76	0.281	0.289*
TRCD	22.09	15.86	4.75	1.45	0.083	0.084
TRY	57.05	13.24	36.98	6.71	0.152	0.153
TRU	38.09	16.08	10.48	11.34	0.437	0.468**
WTAN	17.12	12.58	2.37	2.13	0.195	0.198
Average	**26.51**	**16.51**	**7.49**	**2.47**	**0.125**	**0.128****

The contribution to total risk of the covariance term tends to be relatively small for each of the three periods. In the first five year period, 22 of the trusts have positive covariance and 28 of the trusts have negative covariance. The distribution of correlation coefficients is shown in Figure 4.7. For the average trust in the first period the correlation coefficient is negative (significant at 5% level, two tail test). But note the change of sign in the correlation coefficient for the average trust. For the second and third periods it is positive (in both cases significant at 1% level, two tail test). The distribution of correlation coefficients for the second and third periods are shown in Figures 4.8 and 4.9 respectively.

Table 4.13 is a summary table for the average trust in each of the three sub-periods, with the results in percentage form. The components are expressed as a percentage of the variance of share returns.

Table 4.13: Importance of the three components for the average trust

	Var(navr)	Var(disr)	2 x Covar
1/82 - 12/86	57%	49%	-6%
1/87 - 12/91	67%	21%	12%
1/92 - 12/96	62%	28%	9%

The main features of Table 4.13 are the reduction in the relative importance of discount volatility for the average trust after the first period, and a negative covariance term in the first period followed by a positive covariance term in the second and third periods..

One possible reason for the apparent negative covariance term in the first 5-year period is that share prices were slow to react to changes in underlying NAVs prior to Big Bang, not least because information was not available.

Figure 4.7 - Distribution of correlation coefficients with monthly returns from 1/82 to 12/86

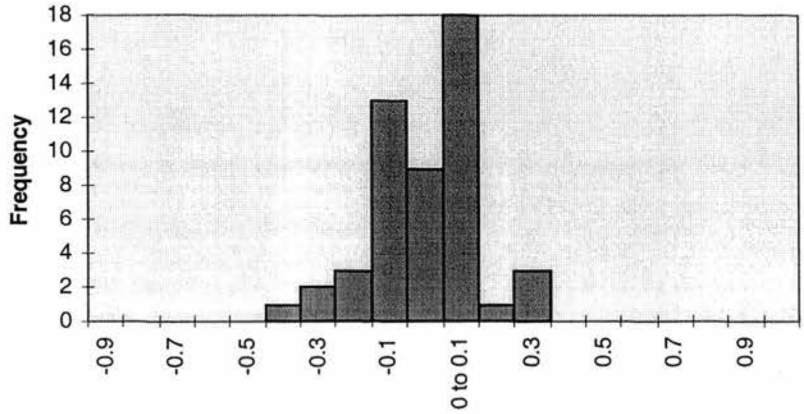


Figure 4.8 - Distribution of correlation coefficients with monthly returns from 1/87 to 12/91

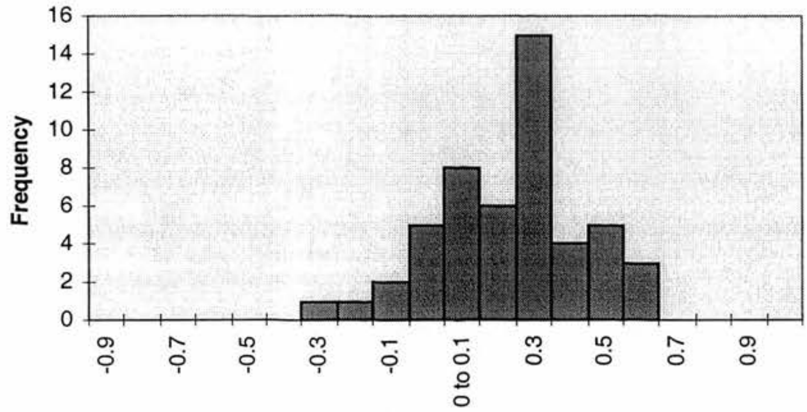
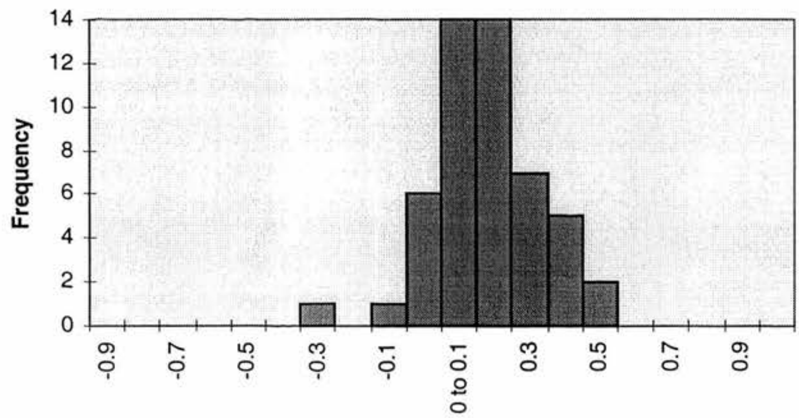


Figure 4.9 - Distribution of correlation coefficients with monthly returns from 1/92 to 12/96



There is strong evidence of a positive correlation coefficient between discount returns and NAV returns since 1987. One possible reasons for this is the indirect capital gains tax effect discussed in 4.5.1. A more likely reason is overreaction to the fundamentals by market participants in the investment trust sector since Big Bang. A positive covariance term is consistent with noise traders 'jumping on the bandwagon' as regards NAV performance.

Private investors (who are noise traders in the Lee *et al* investor sentiment model) became much more aware of investment trusts in the second half of the 1980s. The AITC carried out a successful marketing campaign in 1984/85, savings schemes started to become popular in the mid-1980s, and Personal Equity Plans (PEPs) emerged in 1987. But according to the investor sentiment theory of the discount, the growing importance of private investors (noise traders) should be reflected in a widening discount. This is not observed. The sector average discount narrowed over the period 1/87 - 12/96 (see Figure 2.1).

4.5.6 Effect of the October 1987 crash

October 1987 was an exceptional month and arguably could be distorting the results. In this month, for trusts in the sample, the unweighted average NAV return and discount return were -31% and -4% respectively. The analysis is therefore repeated with the month of October 1987 excluded. Figures for the average trust for both the full period of observation and the five-year period from January 1987 to December 1991 are shown in Table 4.14.

Table 4.14: Results for average trust with month of October 1987 removed

	Var(shr)	Var(navr)	Var(disr)	2xCovar	Corr	Fisher
1/82-12/96	31.51	18.98	11.19	1.34	0.056	0.056**
1/87-12/91	38.90	23.87	11.89	3.09	0.125	0.129**

The differences caused by removing the month of October 1987 from the data set can be observed by comparing the figures in Table 4.14 with the average figures in Table

4.3a (for 1/82-12/96) and Table 4.11 (for 1/87-12/91). As might be expected, variance of share price return, variance of NAV return and variance of discount return are all reduced by the exclusion of October 1987. The average correlation coefficient between NAV returns and discount returns has also been reduced substantially although it is still significantly positive (at the 1% level, two tail test) for both the full period of observation and the five year period from January 1987 to December 1991.

Table 4.15 shows the three components of risk for the average trust with and without October 1987 removed, expressed as a percentage of the variance of share returns.

Table 4.15: Importance of the three components for the average trust (with and without October 1987 removed)

		Var(navr)	Var(disr)	2xCovar
1/82 - 12/96	with 10/87	63%	30%	7%
	without 10/87	60%	36%	4%
1/87 - 12/91	with 10/87	67%	21%	12%
	without 10/87	61%	31%	8%

The reduction in the importance of the covariance term on excluding the month of October 1987 suggests that the positive correlation between discount return and NAV return observed in the last decade or so may be partly related to liquidity, as one of the main features of the October 1987 crash was the difficulty in being able to deal.

The unweighted average discount return for the five trusts specialising in the Far East (including Japan) was -17% in October 1987, a severe 'double whammy' effect. Yet the average NAV return for these five trusts in October 1987 was -32%, very similar to the average of -31% for the total sample. As institutions hold the vast majority of the shares in these trusts, this suggests an institutional investor sentiment effect ('herd instinct') rather than an individual investor sentiment effect.

4.5.7 Sample split into sub-sectors

Most investment trusts in the sample fall into one of three broad categories: international; UK; and geographical. We therefore calculate the average correlation coefficient for each of these sub-sectors. There are 23 international trusts, 12 UK trusts and 12 geographical trusts. Of the remaining three trusts out of the sample of 50, one is a venture capital trust and two are European trusts.

Table 4.16: Average correlation coefficients for sub-sectors.

Period	**International** (23 trusts)	**UK** (12 trusts)	**Geographical** (12 trusts)
1/82 - 12/86	-0.028	0.002	-0.068
1/87 - 12/91	0.236**	0.069	0.203**
1/92 - 12/96	0.118**	0.130**	0.103**
1/82 - 12/96	0.111**	0.053*	0.090**

Table 4.16 gives the figures for the three five-year periods and for the whole 15 year period. Positive and significant correlation coefficients are observed for each of the sub-sectors for both the later 5-year periods, with the exception of UK-invested trusts in the period 1/87 - 12/91 for which the correlation coefficient is positive but not significant. For the early period 1/82 - 12/86, negative correlation coefficients are observed for the international and geographical sub-sectors but they are not statistically significant. For the whole 15 year period, the average correlation coefficient is positive for all sub-sectors. It is significant at the 1% level for geographical and international trusts but only at the 5% level for UK-invested trusts. It is not clear why the 'double whammy' effect is stronger for trusts investing overseas. This is an interesting area for future research.

4.5.8 Suggested decision rules

The analysis suggests that a decision rule based not only on the level of the discount but also based on exploiting the positive covariance term might be successful. For example, trusts on wide discounts would only be purchased if the NAV has risen x% and trusts on narrow discounts would only be sold if the NAV has fallen x%. This would mean less dealing activity in carrying out a particular decision rule but should also result in higher abnormal returns.

If we assume that the investor sentiment theory of the discount is at least partially true, trusts with high discount volatility should stand on high discounts. This suggests that a decision rule based not only on the level of the discount but also on the level of discount volatility might be successful. Thus, trusts on high discounts would only be purchased if their historic discount volatility were below a certain level. Trusts on narrow discounts would only be sold if their historic discount volatility were above a certain level.

4.6 SUMMARY

The results for the entire 15 year period of observation provide strong evidence of excess volatility of share returns compared with NAV returns, and is consistent with the existence of noise traders operating in the market. This is true for monthly, three-monthly and six-monthly returns although the effect reduces as the return interval increases because some of the transient noise is being removed. This excess volatility contradicts the efficient market model although indirect capital gains tax effects (reluctance to sell shares as the market rises) may be partly responsible and time series estimation problems could be distorting the results.

The covariance term is small but has been significantly ***greater*** than zero for the average trust since 1987 which means that discounts have tended to widen (or narrow) when the underlying NAV falls (or rises). This contrasts with the negative covariance term reported by Pontiff (1997). Again, this 'double whammy' effect could be due to noise traders overreacting to changes in the fundamentals (NAVs). The direct contingent capital gains tax problem for US closed-end funds discussed in Section 4.1, which works in the opposite direction, is not relevant to the current UK study.

The results for monthly returns over the entire period of observation show the variance of discount return to be less important than in the Pontiff (1997) study of US closed-end funds but still represents 30% of the variance of share returns for the average UK trust. There is, however, considerable cross-sectional variation in the magnitude of this discount volatility and there seems to be persistence in the relative importance of discount volatility for a given trust. As the return interval is increased, discount volatility becomes less important. This is because the trust share price and underlying NAV time series are cointegrated implying negative autocorrelation of discount movements.

There is clear evidence of a reduction in discount volatility over the 15 year period of observation. This may be due both to an increase in the number of discount anomaly traders (following the success of decision rules in the past) and a reduction in transaction costs. It suggests that decision rules based purely on discount movements may not generate excess returns in future. However, assuming that the positive covariance between discount returns and NAV returns persists, decision rules based partly on this 'double whammy' effect and partly on discount movements might still lead to excess profits.

CHAPTER 5 - CROSS-SECTIONAL VARIATION IN DISCOUNT VOLATILITY

5.1 INTRODUCTION

We have seen that discount volatility is generally an important component of the total risk for UK investment trusts, but the magnitude of this discount volatility varies widely across the sector. In this chapter, we test various potential sources of discount volatility by examining whether they can explain the cross-sectional variation in discount volatility for UK investment trusts. This may help to explain why discounts fluctuate so widely over time.

We again use the following notation:

P_t = share price of investment trust at time t

A_t = net asset value per share at time t

Then 'discount return' for period t is defined as (see 3.4.1):

$$\log_e(1+R_t^D) = \log_e(P_t / P_{t-1}) - \log_e(A_t / A_{t-1})$$
$$= \log_e(1+R_t^P) - \log_e(1+R_t^A)$$

where $\log_e(1+R_t^P)$ is the share price return in period t

and $\log_e(1+R_t^A)$ is the NAV return in period t.

We define *discount volatility* to be the standard deviation of monthly discount return.

The marked difference in the ownership of US closed-end funds and UK investment trusts is relevant to the analysis in this chapter. Typically, individuals hold a much higher percentage of the equity of US closed-end funds compared with UK investment trusts and there is considerably more variation in the importance of individual shareholders across the UK investment trust sector. It should therefore be relatively easy in the case of UK investment trusts to test whether discount volatility increases with the proportion of shares held by individuals who, according to the Lee *et al* (1991a) investor sentiment theory, are the 'noise traders' responsible for discount volatility. Their irrational activity creates (systematic) risk and informed rational investors who have limited time horizons fear to trade against them. The more individuals (noise traders) who are shareholders, the greater the discount volatility we would expect.

One possible problem in studying the variation in discount volatility across the UK investment trust sector is that the timing of NAV reporting is not consistent for different trusts. NAVs may be published daily, weekly or monthly. Monthly NAVs are generally not known until up to ten days after the month-end.[1] However, Datastream and Reuters estimate NAVs for most investment trusts on a daily basis. This means that month-end share prices should be largely based on accurate NAV data, thereby minimising inaccuracies from this source in calculating discounts and discount volatility. Hoskins (1994) does not suffer from this problem at all in his cross-sectional analysis of US discount volatility. US closed-end funds have reported NAVs to pricing agencies on a weekly basis since 1965 and Hoskins uses weekly data in calculating discount volatility.

As discussed in Chapter 4, there is a discontinuity in the share price when the share goes ex-dividend, as is the case for any share. But there is no discontinuity in the corresponding NAV because this is calculated excluding the revenue account for the current year. As a result, there is a discontinuity in the discount when the share goes ex-dividend. This has a slight artificial impact on discount volatility, normally but not always tending to increase it. The effect tends to be greater for higher yielding trusts. In 4.4.1 we investigated the effect of adjusting share prices to eliminate this discontinuity before calculating the components of total risk. Table 4.1 shows that the adjustment makes very little difference to discount volatility (the square root of the variance of discount return). We will therefore not worry about adjusting for share price discontinuities in this chapter.

Again, as discussed in Chapter 4, some trusts have warrants in issue. If so, it is normal practice in the investment trust industry to make adjustments to NAV on a per share basis by treating warrants as exercised if dilution of NAV would occur, to

[1] Datastream 'corrects' month-end NAVs when official monthly NAVs are reported whereas Reuters does not adjust the month-end estimates retrospectively.

give a 'fully diluted' figure. Discounts are then calculated by relating share price to fully diluted NAV. An alternative approach is to calculate discounts on a 'package' basis. In 4.4.2 we calculated the components of total risk using both approaches separately for those trusts in the sample with warrants in issue but it did not make any qualitative difference to the results. In particular, Table 4.2 shows that discount volatility is hardly affected. We will therefore simply use the more common 'fully diluted' method in this chapter.

We identify possible trust attributes that may influence discount volatility in Section 5.2. The data to be used in the analysis is then discussed in Section 5.3. In Section 5.4, we carry out regressions to explain the cross-sectional variation in discount volatility using the identified trust attributes as explanatory variables and draw conclusions from the results.

5.2 TRUST ATTRIBUTES THAT MAY INFLUENCE DISCOUNT VOLATILITY

In this section we discuss the choice of trust attributes to be used as explanatory variables in the cross-sectional analysis. Trust share turnover and standard deviation of NAV return are by far the most important attributes for explaining discount volatility in Hoskins' study of US closed-end funds (see 3.4.6) and are therefore included in our study. All the other attributes in the Hoskins study - R-squared with the market, portfolio turnover, dividend yield, underlying net assets, expense ratio, share growth and average share price - are not significant as explanatory variables in his regression equation, even at the 5% level. Nevertheless, two of the remaining six variables chosen in our study, Ln(market value) and Ln(unadjusted share price), have close equivalents in the Hoskins study. Precise definitions of the eight trust attributes chosen as explanatory variables for the cross-sectional analysis are given in Table 5.1 at the end of Section 5.2.

We consider separately those factors that influence discount volatility through share price returns and those factors that influence discount volatility through NAV returns. Factors in the former category are related to the question of market efficiency/

investor irrationality. Factors in the second category are concerned with miscalculation of NAV or the method of calculating discount to NAV. There are analogies here with two of the broad categories of factors that have been proposed to explain the main element of the discount puzzle (see Sections 3.2 and 3.3).

5.2.1 Factors related to share price returns

Trust share turnover (number of shares traded divided by number of shares outstanding), sometimes known as trading velocity, reflects the level of information arriving at the market, and information may be regarded as the driving force for share price movements. But the extent to which this is translated into discount movements depends on the ability of discount anomaly traders to carry out their activities. Hoskins finds share turnover to be a very significant positive influence on US closed-end fund *weekly* discount volatility, contrary to what he expected, but his explanation for the result relies on the fact that closed-end funds with high share turnover tended to stand on large premiums.[2] This is not the case with the 59 UK investment trusts which form the sample to be analysed in this chapter (see Section 5.3). Of the ten trusts with the highest share turnover, only one reached a premium of more than 12% during the five year period of observation, with the other nine trusts each trading at a discount on average over the period.

Low marketability of the trust shares makes discount anomaly trading less profitable. If a discount anomaly opportunity exists, there is only a small potential profit because a relatively small order will correct the pricing anomaly. Thus, the lower the marketability of the trust shares, the greater the discount trading range, and hence the higher the discount volatility. Market value of a trust is often taken as a rough proxy for marketability (see, for example, the London Business School Risk Measurement Service). We take *ln(market value)* as the chosen explanatory variable so that the same percentage difference cross-sectionally in market value at different levels of market value has the same effect in the analysis.

[2] Hoskins argues that shares could no longer be borrowed for shorting once the funds stand on large premiums and the normal activity of traders adding liquidity to the market would be shut down. This would greatly reduce the liquidity for fund shares, and high trading volume could then create very large discount volatility.

There may be a tendency for lower share prices to be associated with larger bid-offer spreads (as a percentage of share price), which directly increases dealing costs associated with discount anomaly trading. This implies that trusts with lower share prices will tend to have higher discount volatility. The share price to be used as an explanatory variable must be 'unadjusted', that is not adjusted for subsequent capital changes. We should also take logarithms so that the same percentage difference cross-sectionally in share price at different levels of share price has the same effect in the analysis. So we take *ln(unadjusted share price)* as an explanatory variable.

Unadjusted share price may be positively correlated with market value. Both are related to bid-offer spread as a percentage of share price. At an appropriate point in the analysis it may be appropriate to drop one of these two variables.

According to the investor sentiment theory, small (individual) investors deal on 'noise' rather than on the fundamentals. Rational arbitrageurs fail to offset fully the discount anomalies created by the irrational whims of individual investors because such arbitrageurs have finite time horizons implying that their activities are risky and therefore limited. As there is considerable variation in the importance of individual shareholders across the UK investment trust sector, it is easy to test whether discount volatility is related to the *percentage of shares held by individuals***.**

There is a different type of investor sentiment that may be relevant to discount volatility, namely 'UK market investor sentiment'. Investment trust shares are traded in the UK and may therefore be subject to investor sentiment that is specific to the UK. This UK specific sentiment will tend to increase discount volatility if the underlying assets are held overseas. But if the underlying assets are held in the UK, there will be a cancelling out effect and no consequent influence on discount volatility. Furthermore, discount volatility for funds with foreign assets may partly reflect factors that preclude costless cross-border transactions: official and unofficial barriers to capital movements, transaction costs, time to complete transactions and time mismatch in trading hours. We therefore include *percentage of underlying assets in the UK* as an explanatory variable.

5.2.2 Factors related to NAV return

Taking advantage of discount anomalies without exposure to movements in the underlying market(s) is difficult if the underlying net assets are volatile because this makes hedging the underlying net assets more difficult. But having a good hedge is all the more important in this situation because a poor hedge will translate into larger gains or losses. So volatile underlying net assets makes hedging difficult but also losses (or gains) from not hedging properly tend to be large. We therefore include *standard deviation of NAV return* as an explanatory variable.

Another possible reason for the influence of standard deviation of NAV return on discount volatility could be staleness of the trust share prices themselves. Such staleness would imply sluggish share price response to NAV movements, so the more volatile the NAV return the greater the discount volatility. But as discount volatility is calculated monthly in our study rather than, say, weekly as is common in US studies, this is likely to be of minor importance.

Standard deviation of NAV return will depend partly on the trust's area of specialisation, as defined by the five NWS investment trust sub-sectors - International, UK, Geographical, Europe and Venture Capital (see Appendix 2A). International trusts should have relatively low standard deviation of NAV return. Correlations between the returns from shares held in the 'world' market are generally less than those between the returns from shares confined to a particular domestic equity market such as that of the UK, even when foreign-exchange risk is fully borne by the fund[3] (see, for example, Solnik (1996)). So international diversification reduces the standard deviation of NAV return. Geographical specialists will tend to have high standard deviation of NAV return because they are not diversified to the same extent, their underlying markets are often volatile (e.g. emerging markets) and currency movements increase NAV volatility. Venture capital trusts, which have a high proportion of unquoted assets, will tend to have low standard deviation of NAV return because valuations of the unquoted assets have varying degrees of staleness which tends to smooth NAV returns.

[3] However, currency exposure can be managed independently of the underlying portfolio and this may be carried out with the aim of boosting returns.

As changes in underlying NAV are an important flow of information, it is likely that trusts with volatile underlying net assets will attract trading activity[4] so standard deviation of NAV return will be positively correlated with the first explanatory variable in 5.2.1, trust share turnover, leading to a possible multicollinearity problem.

Gearing will influence discount volatility indirectly through its influence on the standard deviation of NAV return. But gearing also directly affects discount volatility because it reduces the denominator (NAV) in the discount to NAV calculation. This can be seen as follows:

Discount to NAV = (NAV - Value of equity)/NAV

= (Value of assets - Value of debt - Value of equity)/NAV

Therefore,

Discount to NAV = (Value of assets - Total value of the firm)/NAV (5.1)

If we hold the value of the underlying assets constant and we assume that the level of gearing has no influence on the total market value of the individual firm, in line with the Modigliani and Miller (1958) and Miller (1977) proposition, it follows that the numerator of the right hand side of equation (5.1) will not depend on the level of gearing. But the denominator (NAV) will be lower for higher levels of gearing. Thus, movements in the difference between the value of the underlying assets and the total market value of the firm (the discount) will be exaggerated with higher levels of gearing because discount to NAV is expressed as a proportion of NAV rather than as a proportion of total market value. It follows that gearing tends to increase discount volatility.

Also, the true value of debt will be different from the nominal value of debt used in the NAV calculation which, assuming market efficiency, will further add to discount volatility depending on the level of gearing.

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[4] This trading activity may be based on the fundamentals rather than discount anomaly trading.

Directors' valuations of unquoted investments[5] may be historic to some extent, only changing when 'something happens', such as a share stake changing hands. This is particularly relevant to venture capital trusts which invest mainly in unquoted companies. As explained in 3.4.2, if the underlying portfolio contains shares whose prices have different degrees of staleness, the NAV time series acts like a moving average of past 'true' prices and will be artificially smooth as a result. This has a direct effect on discount volatility as it reduces the correlation between share price returns and NAV returns. We therefore include *percentage of underlying assets which are unquoted* as an explanatory variable.

It has already been noted that valuations of the unquoted assets have varying degrees of staleness which tends to smooth NAV returns, so there may be negative correlation between the percentage of underlying assets which are unquoted and the standard deviation of NAV return, a trust attribute which has already been identified as a possible explanatory variable.

[5] British Venture Capital Association guidelines are followed by most trusts.

Table 5.1: Definitions of trust attributes chosen as explanatory variables

Trust share turnover	Average over months 1/92 to 12/96 of (no of shares traded in month divided by average no of shares outstanding in that month)
Standard deviation of NAV return	Standard deviation of monthly undiluted NAV return over the months 1/92 to 12/96
Gearing	1/2*(actual gearing at 31/12/91 + actual gearing at 31/12/96) where actual gearing is the ratio of total assets (less fixed interest and cash assets) to shareholders' funds
Ln(market value)	Natural logarithm of the average over months 12/91 to12/96 of the month-end market value of the trust
Ln(unadjusted price)	Natural logarithm of the average over the months 12/91 to 12/96 of the month-end share price (unadjusted for subsequent capital changes)
% of underlying assets which are unquoted	1/2*(% assets unquoted at 31/12/91 + % assets unquoted at 31/12/96)
% of shares held by individuals	Percentage of the share capital of the investment trust held by individual investors (1994, where possible)[6]
% of underlying assets in the UK	1/2*(% of assets in UK on 31/12/91 + % of assets in UK on 31/12/96)

[6] It is debateable as to whether 'nominee' accounts should be included with individuals or institutions. After consultation with industry practitioners, it was decided to treat half of the shares held in nominee accounts as being held by individuals and half as held by institutions.

5.3 DATA

The sample consists of the 59 UK investment trusts in continuous operation over the five years from 1 January 1992 to 31 December 1996 for which share trading volume data are available on Datastream.[7] These tend to be the largest trusts in the sector and they were all constituents of the FT-SE Actuaries All Share Index on 31 December 1996. Trusts created during the period of observation are excluded. The sample of trusts is detailed in Appendix 5.

Table 5.2 gives data sources for all variables in the analysis. Table 5.3 shows the actual values for the dependent variable, discount volatility, and for all the explanatory variables. A number of relevant points can be made from the study of Table 5.3:

a) The average value for discount volatility (column 3) is 3.25% which compares with an average value for the standard deviation of NAV return (column 5) of 4.40%. These are a little higher than the corresponding average figures (2.74% and 4.06% respectively) for the sample in Chapter 4 over the same period, reflecting the fact that the sample in this chapter includes a higher proportion of geographical specialists.

b) Discount volatility varies widely across the sample, ranging from 7.22% for Dartmoor Investment Trust to 1.60% for Kleinwort Overseas Investment Trust. (Column 3).

c) International trusts tend to have relatively low standard deviation of NAV return, geographical specialists tend to have relatively high standard deviation of NAV return, and the two venture capital trusts have low standard deviation of NAV return. (Column 5). This is consistent with points made in 5.2.2.

d) There is little variation in the level of gearing across the sector. Only one trust in the sample, Dartmoor Investment Trust, has a high level of gearing (Column 6). (This trust also has the highest standard deviation of NAV return and the highest discount volatility.)

Figure 5.1 illustrates the distribution of the dependent variable, discount volatility, and Figures 5.2 to 5.9 illustrate the distributions of the explanatory variables.

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[7] Bloomberg was used to obtain missing values in the data.

Table 5.2: Data sources for variables in the cross-sectional regression analysis

Discount volatility	Datastream
Trust share turnover	Datastream Bloomberg
Standard deviation of NAV return	Datastream
Gearing	NatWest Securities, Daily NAV Service, Year-end 1996 County NatWest WoodMac, Daily NAV Service, Year- end 1991
Ln(market value)	Datastream
Ln(unadjusted price)	Datastream
% of underlying assets which are unquoted	NatWest Securities, Daily NAV Service, Year-end 1996 County NatWest WoodMac, Daily NAV Service, Year-end 1991
% of shares held by individuals[8]	NatWest Securities, Investment Trust Annual, 1994-95 AITC Investment Trust Directory, Summer 1994 AITC Investment Trust Index, 1992 NatWest Securities, Shareholders Over 3%, April 1997
% of underlying assets in the UK	NatWest Securities, Daily NAV Service, Year-end 1996 County NatWest WoodMac, Daily NAV Service, Year-end 1991

[8] There was some difficulty in obtaining the percentage of shares held by individuals for some trusts. In three cases, Abtrust New Dawn, Templeton Emerging Markets and Foreign & Colonial German, an estimate had to be made on the basis of the little information that was available on shareholdings.

Table 5.3: Values for dependent variable and explanatory variables

(1) trust	(2) sector	(3) discvol	(4) turnover	(5) stdevnavr	(6) gearing	(7) ln(MV)	(8) ln(UPrice)	(9) %U/Q	(10) %individs	(11) %UK
aot	internatnl	0.0164	0.0277	0.0357	105	6.1216	5.9887	3.5	9.2	41
bnkr	internatnl	0.0245	0.0372	0.0390	101.5	5.6207	5.1825	0.5	30.28	47.5
bti	internatnl	0.0205	0.0108	0.0363	101	5.0894	5.7579	3.5	16.1	54.5
bset	internatnl	0.0236	0.0318	0.0362	131	5.8280	4.5494	1	39.2	72.5
btem	internatnl	0.0205	0.0252	0.0366	99.5	4.8211	4.4183	7	4.68	57.5
edin	internatnl	0.0185	0.0231	0.0367	110	6.7498	5.6818	2	49.6	77
ensc	internatnl	0.0281	0.0201	0.0384	102	5.2075	4.7350	4.5	16.5	48.5
fcs	internatnl	0.0315	0.0357	0.0371	106	5.0260	5.0306	7.5	34.8	49
frcl	internatnl	0.0194	0.0391	0.0375	108	7.2081	5.1820	4	45.9	39.5
mnks	internatnl	0.0173	0.0208	0.0397	95.5	5.9503	6.2040	0.5	18.1	37.5
myi	internatnl	0.0255	0.0290	0.0379	101.5	5.9685	5.8039	24	53.1	48
rcp	internatnl	0.0438	0.0255	0.0261	105.5	5.7005	5.1114	29.5	16.56	24.5
scam	internatnl	0.0181	0.0266	0.0291	111.5	5.8277	5.0230	5.5	46.1	64
scea	internatnl	0.0211	0.0215	0.0377	113.5	6.2356	4.3894	7.5	18.2	52.5
scin	internatnl	0.0167	0.0273	0.0362	102	6.3774	5.3952	4.5	36.2	47.5
smt	internatnl	0.0203	0.0240	0.0397	106.5	6.6654	5.3825	0.5	23.3	51.5
sts	internatnl	0.0218	0.0221	0.0382	112.5	5.6021	4.4379	1	62.1	69
tru	internatnl	0.0324	0.0296	0.0401	107	5.7875	5.2098	5	12.44	64
wtan	internatnl	0.0154	0.0326	0.0372	102	6.6989	5.3928	2.5	12.5	60
dit	uk	0.0722	0.0627	0.0781	194.5	3.6389	4.7539	0.5	52.9	100
fmn	uk	0.0249	0.0330	0.0345	101	5.9980	5.6299	15	15.1	80.5
gvs	uk	0.0232	0.0439	0.0486	104.5	5.5398	5.5584	1	8.4	88
iei	uk	0.0705	0.0577	0.0609	129.5	3.8784	4.5639	15.5	19.1	91.5
mrch	uk	0.0232	0.0298	0.0427	105.5	5.5605	5.5376	1	39.5	91.5
mgs	uk	0.0416	0.0328	0.0413	87.5	4.5238	4.7844	1	12	96.5
mge	uk	0.0326	0.0380	0.0402	98	3.7150	4.8781	0	91.9	98
mut	uk	0.0212	0.0239	0.0359	95	5.6162	5.7913	10.5	61.2	84.5
smc	uk	0.0398	0.0775	0.0436	98.5	3.8471	4.7588	0	20	100
tmpl	uk	0.0247	0.0300	0.0382	97.5	5.2479	5.8043	4.5	42.2	95
thrg	uk	0.0489	0.0426	0.0464	126.5	5.3583	4.2610	19	23.5	98.5
trcd	uk	0.0218	0.0278	0.0398	109	5.6764	4.9835	0	60.5	100
try	uk	0.0608	0.0515	0.0364	116.5	4.7008	3.3660	22.5	12.4	84.5
vin	uk	0.0333	0.0211	0.0311	133.5	3.8101	4.6705	37.5	18.1	99
abd	geograph	0.0383	0.0581	0.0617	95.5	4.2508	5.2380	0	31.8	4
amts	geograph	0.0343	0.0500	0.0377	95.5	5.3580	5.5500	3.5	24.2	6.5
efm	geograph	0.0338	0.0311	0.0746	104.5	5.2881	4.2338	0.5	12	1.5
fam	geograph	0.0319	0.0530	0.0422	99	5.2807	5.6372	6.5	15.3	3
fem	geograph	0.0407	0.0254	0.0618	86.5	4.8158	4.9442	1	17.8	10.5
ffe	geograph	0.0284	0.0307	0.0654	118	6.1202	5.7068	2	12.8	0
flmj	geograph	0.0360	0.0677	0.0680	111	5.8286	5.3792	3	12.1	2
fov	geograph	0.0189	0.0293	0.0368	97.5	5.9143	5.6303	4.5	14.6	4.5
fct	geograph	0.0590	0.0607	0.0617	107.5	4.8698	4.5463	15.5	0.6	10
fcp	geograph	0.0307	0.0640	0.0497	99	5.7875	5.4642	1.5	34.5	1
gtja	geograph	0.0516	0.0535	0.0551	97.5	4.9062	5.3779	2	12	1
gtm	geograph	0.0439	0.0712	0.0720	108.5	4.2635	4.7038	0	12	0.5
gor	geograph	0.0281	0.0588	0.0561	120	6.3301	5.6746	4.5	14.3	1
kos	geograph	0.0160	0.0319	0.0379	103	5.2061	5.4264	7	18.7	14
msm	geograph	0.0259	0.0279	0.0517	109	5.3859	5.9708	7	51.9	14
oit	geograph	0.0246	0.0286	0.0378	97	4.8111	5.7826	0.5	15.7	0.5
tem	geograph	0.0456	0.0570	0.0595	89.5	5.8655	5.4133	0	34.7	9
trv	geograph	0.0441	0.0612	0.0705	102.5	4.8468	4.7808	4	31.2	1
fev	europe	0.0340	0.0440	0.0346	90.5	4.3336	5.0438	0.5	11	25
fut	europe	0.0312	0.0389	0.0404	101	5.3014	5.7279	9.5	18.4	5
fef	europe	0.0431	0.0503	0.0376	106.5	3.7712	4.5068	1.5	15.8	0
fcg	europe	0.0390	0.0597	0.0390	91.5	3.8254	4.7592	0	19.2	0.5
fcu	europe	0.0279	0.0400	0.0392	108	4.9186	5.4202	1	70.51	4
klc	europe	0.0256	0.0237	0.0364	109.5	5.1377	5.3483	17	11.6	55
elta	vencap	0.0579	0.0339	0.0263	103.5	6.2405	5.7011	64.5	8	62.5
fcet	vencap	0.0532	0.0153	0.0321	98	4.1718	4.2332	65.5	6.8	74

Figure 5.1 - Distribution of discount volatility

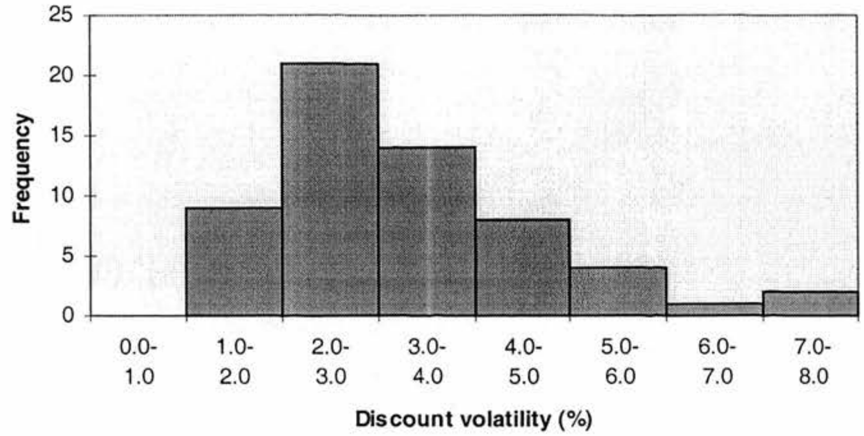


Figure 5.2 - Distribution of trust share turnover

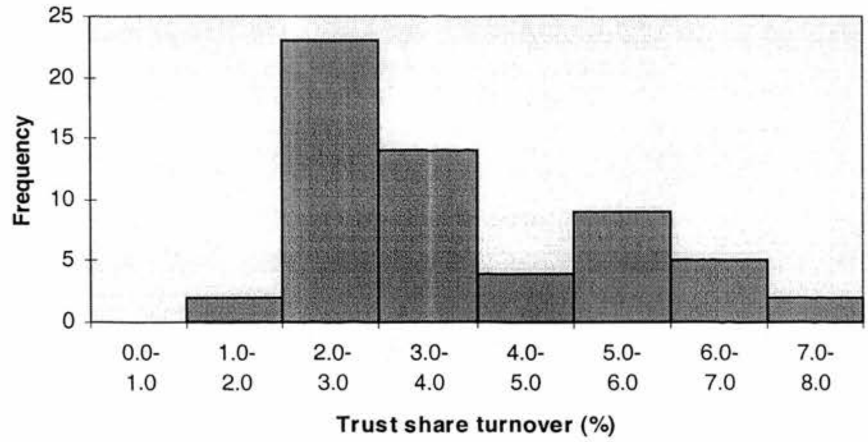


Figure 5.3 - Distribution of standard deviation of NAV return

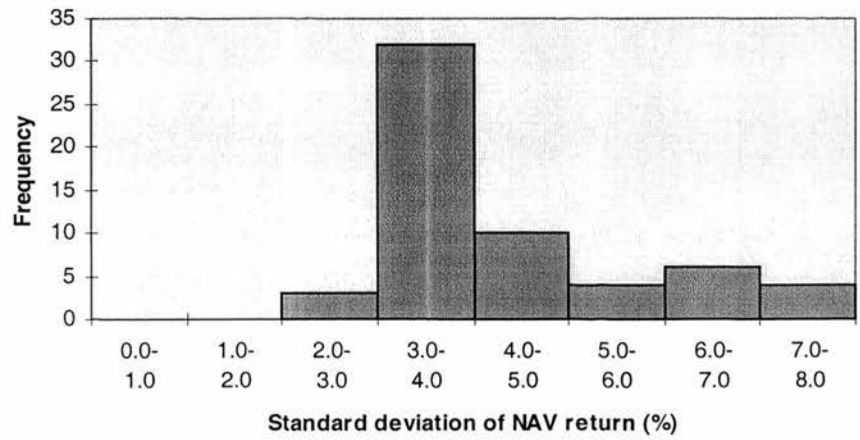


Figure 5.4 - Distribution of gearing

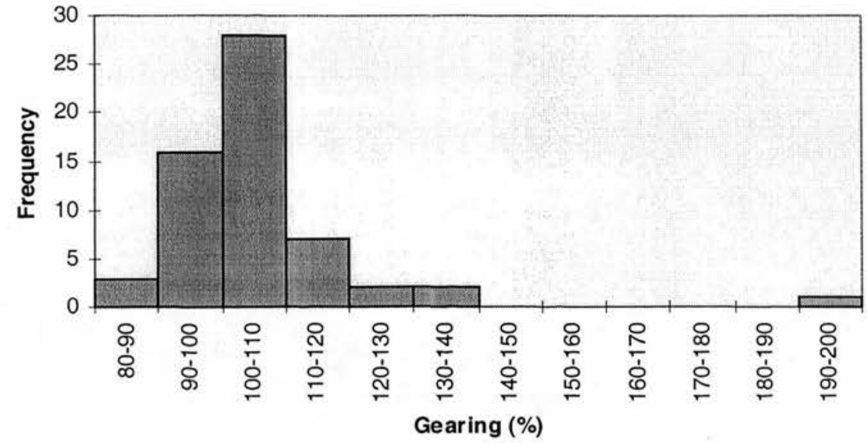


Figure 5.5 - Distribution of ln(market value)

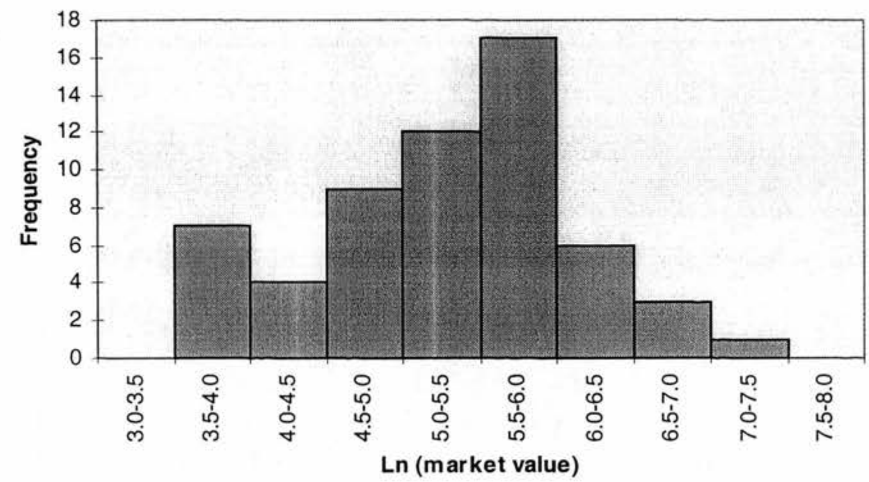


Figure 5.6 - Distribution of ln(unadjusted price)

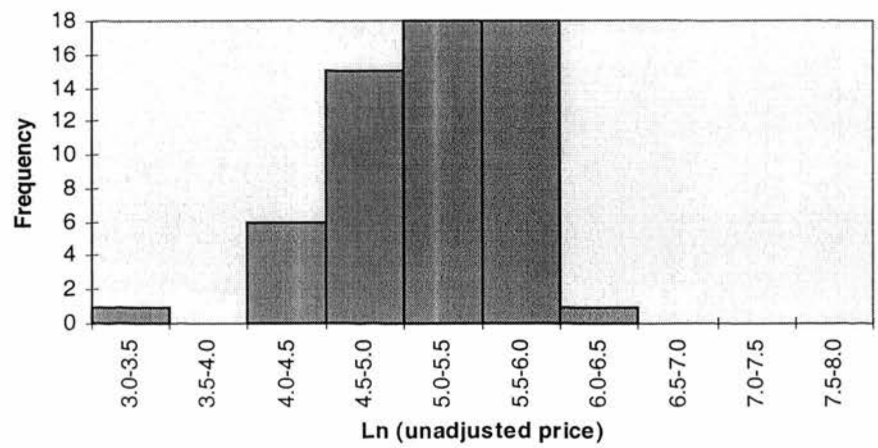


Figure 5.7 - Distribution of percentage of underlying assets which are unquoted

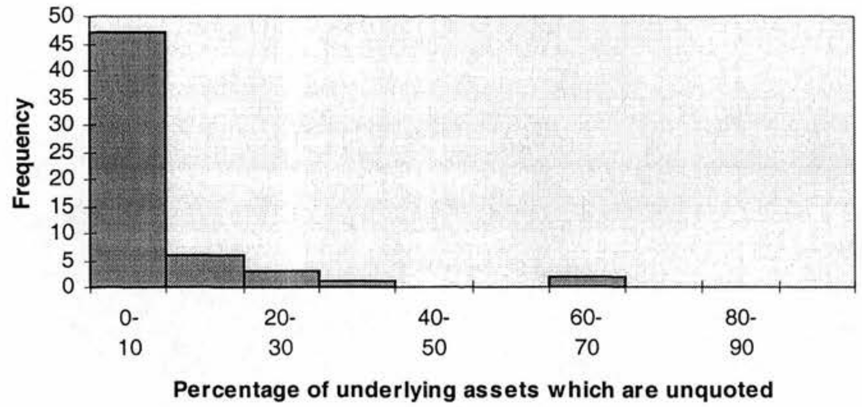


Figure 5.8 - Distribution of percentage of shares held by individuals

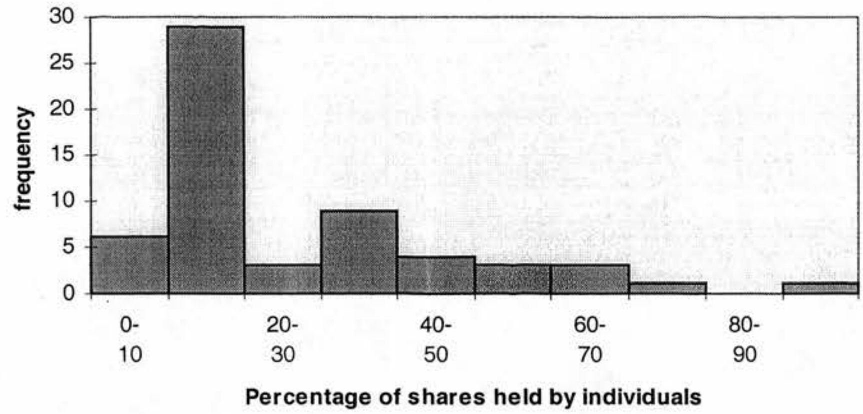
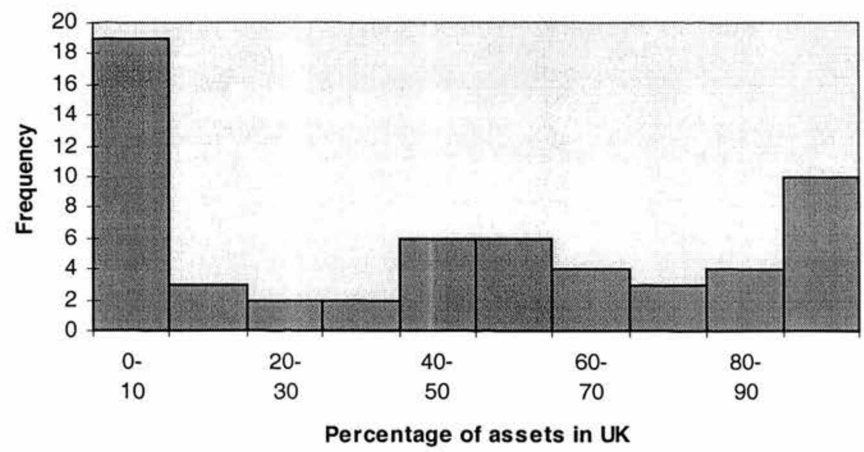


Figure 5.9 - Distribution of percentage of assets in UK



5.4 REGRESSION ANALYSIS

We now carry out regressions to explain the cross-sectional variation in discount volatility. Table 5.4 shows cross correlations for the explanatory variables. It confirms a number of points made in Section 5.3. There is high positive correlation between trust share turnover and standard deviation of NAV return, and also between ln(market value) and ln(unadjusted price). Standard deviation of NAV return is positively correlated with gearing and negatively correlated with percentage of underlying assets which are unquoted.

Table 5.4: Cross correlations of explanatory variables

	turnover	stdevnavr	gearing	ln(mv)	ln(uprice)	%u/q	%individs
turnover							
stdevnavr	0.58						
gearing	0.14	0.30					
ln(mv)	-0.36	-0.23	-0.13				
ln(uprice)	-0.18	-0.13	-0.27	0.47			
% u/q	-0.23	-0.34	0.07	-0.05	-0.17		
% individs	-0.10	-0.07	0.16	0.04	0.11	-0.24	
% UK	-0.30	-0.34	0.33	-0.07	-0.24	0.23	0.27

Table 5.5 shows the expected signs for correlations between discount volatility and each of the explanatory variables, together with the reasoning for these expected signs.

Table 5.5: Expected signs for correlations between explanatory variables and discount volatility

Explanatory variable	Expected sign	Reasoning
Trust share turnover	Positive	Trust share turnover is a proxy for information flow
St dev of NAV return	Positive	Standard deviation of NAV return proxies for both the ability and the need to hedge underlying net assets from the discount anomaly trader's viewpoint
Gearing	Positive	Gearing exaggerates discount movements because the discount is expressed as a percentage of NAV
Ln (market value)	Negative	The higher the market value the more marketable the trust shares and the narrower the discount trading range
Ln(unadjusted price)	Negative	Lower priced shares tend to have larger bid-offer spreads which increases dealing costs associated with discount anomaly trading
Percentage unquoted	Positive	Valuations of unquoted assets tend to be historic which reduces the correlation between share price returns and NAV returns
Percentage individuals	Positive	According to the individual investor sentiment theory, individuals are irrational noise traders who deal on the basis of sentiment rather than the fundamentals
Percentage UK	Negative	The more underlying assets held in the UK, the less impact UK specific sentiment will have on the discount as there is a cancelling out effect

Table 5.6 shows the correlation coefficients between discount volatility and each of the explanatory variables. The signs are as expected for the first six explanatory variables but are opposite to that expected for the last two explanatory variables, namely percentage of shares held by individuals and percentage of underlying assets in the UK. The magnitude of the correlation coefficients between discount volatility and these last two explanatory variables are, however, fairly low. The last two variables suggest little impact of investor sentiment (individuals and UK specific, respectively) on discount volatility. We should be cautious about drawing conclusions, however, as we are looking here at correlations in isolation of one another.

Table 5.6: Correlations between explanatory variables and discount volatility

Explanatory variables	Correlation with discount volatility
Trust share turnover	0.55
St dev NAV return	0.44
Gearing	0.35
Ln(market value)	-0.58
Ln(unadjusted price)	-0.50
Percentage unquoted	0.38
Percentage individuals	-0.21
Percentage UK	0.03

Table 5.7 shows the results of the multiple regression of discount volatility on the eight explanatory variables (Regression (1)). The signs for the regression coefficients are the same as the corresponding correlation coefficients in Table 5.6. The t-statistics, however, indicate that some of the explanatory variables are far more significant than others. Trust share turnover, standard deviation of NAV return, ln(market value) and percentage of underlying assets which are unquoted all have t-statistics which are significant at the 0.5% level (two-tail test). Ln(unadjusted price) has a t-statistic of -1.80 but Table 5.4 shows that this variable is highly correlated with ln(market value), with a correlation coefficient of 0.47, indicating possible multicollinearity in the multiple regression.

Table 5.7: Regressions of discount volatility on explanatory variables

	Regression (1)		*Regression(2)*	
Trust attribute	**Coeff**	**t-statistic**	**Coeff**	**t-statistic**
Constant	0.03362	2.51	0.02817	3.35*
Trust share turnover	0.32392	4.32*	0.31570	4.15*
St dev NAV return	0.33962	3.31*	0.37583	3.99*
Gearing	0.00007	0.99		
Ln (market value)	-0.00421	-3.32*	-0.00543	-4.61*
Ln(unadjusted price)	-0.00349	-1.80		
Percentage unquoted	0.00052	6.61*	0.00058	7.72*
Percentage individuals	-0.00003	-0.47		
Percentage UK	0.00002	0.64		
	Adjusted R-square = 0.76		Adjusted R-square = 0.74	

* significant at the 0.5% level (two-tail test)

Gearing, percentage of shares held by individuals and percentage of underlying assets in the UK have t-statistics of 0.99, -0.47 and 0.64 respectively. It is no surprise that the t-statistic for gearing is not significant; we have already noted that there is little variation in the level of gearing across the sample. What is perhaps surprising is that the t-statistics for the other two variables, which both relate to investor sentiment, are not significant. This is an important result. It suggests that one of the main parts of the discount puzzle - that discounts fluctuate widely over time - cannot be explained by appealing to arguments concerning investor sentiment.

Table 5.7 also shows the results of a regression of discount volatility on the four most significant explanatory variables only (Regression (2)), with the other four original explanatory variables excluded. The adjusted R-square is 0.74 and the constant together with the four explanatory variables each have t-statistics which are significant at the 0.5% level. Note the stability of the coefficients and t-statistics across the regressions. It should be remembered from Table 5.4, however, that there is a correlation coefficient of 0.58 between trust share turnover and standard deviation of NAV return, so the coefficients for these variables may be unreliable due to multicollinearity.

The Belsley condition index for the four variables is 1,534, much greater than 20, confirming that multicollinearity is a problem. On examination of the variance proportions, the variables causing most collinearity are trust share turnover and standard deviation of NAV return, as expected.

The t-statistic for percentage of assets which are unquoted is very high at 7.72. It is interesting to note therefore that if the two venture capital trusts, Electra and Foreign & Colonial Enterprise, which have by far the highest proportion of unquoted assets among trusts within the sample, are removed from the data set, the t-statistic reduces to 4.74. But this is still significant at the 0.5% level and the regression coefficient for this variable is roughly unchanged at 0.00059 (compared with 0.00058). Given the relatively little variation in the proportion of underlying assets which are unquoted across the sector, however, further investigation is necessary.

To test the stability of the regression coefficients in Regression (2), we now split the period of observation into two equal sub-periods, 1/92 to 6/94 inclusive and 7/94 to 12/96 inclusive, and carry out regressions for these two 30 month periods separately. Discount volatility (dependent variable) together with trust share turnover and standard deviation of NAV return (explanatory variables) are all on average much higher in the first sub-period compared to the second sub-period. The results are given in Table 5.8. Note that all four explanatory variables have t-statistics which are significant at the 0.5% level for the regressions in respect of both 30 month periods, as was the case for the full five year period. It is clear, however, that the coefficient for percentage of underlying assets which are unquoted is unstable, being 0.00080 in the first period and much lower at 0.00020 in the second period.[9] Nevertheless, it is still significant in the second period.

[9] Figure 5.7 shows that there is little cross-sectional variation in the 'percentage of underlying assets which are unquoted' explanatory variable.

Table 5.8: Regressions of discount volatility on explanatory variables

	Period 1/92 to 6/94		Period 7/94 to 12/96	
Trust attribute	**Coefficient**	**t-statistic**	**Coefficient**	**t-statistic**
Constant	0.03011	2.51	0.03483	5.38*
Trust share turnover	0.23121	2.82*	0.22526	3.45*
St dev NAV return	0.49108	4.04*	0.26383	3.38*
In(market value)	-0.00667	-4.17*	-0.00494	-5.21*
Percentage unquoted	0.00080	7.37*	0.00020	3.42*
	Adjusted R-square = 0.70		Adjusted R-square = 0.61	

To test the equality of the coefficients in the two equations, a Chow test is carried out. It confirms that the regression equation in the second period is different from that in the first period. The null hypothesis of equality of coefficients in the two equations is rejected (significance of less than 0.05%).

When all eight explanatory variables are included in the regressions, each of the four explanatory variables in Table 5.8 are significant at the 0.5% level for both 30 month periods. The other four explanatory variables are not significant at this level, although Ln(unadjusted price) is significant at the 1% level in the second period. In particular, *percentage of shares held by individuals* is not significant in either period (t-statistics -0.50 and -0.49) and *percentage of underlying assets in the UK* is also not significant in either period (t-statistics 0.30 and -0.02). This confirms the earlier observation that both small investor sentiment and UK specific investor sentiment have little impact on discount volatility.

How does the above analysis help to explain discount volatility? The analysis suggests that the main driving forces for discount movements are: new information (including changes in NAV) being impounded in share prices; and volatility of NAV returns. Discount anomaly traders try to take advantage of discount anomalies that may result but they have limited time horizons and their activities are restricted by transaction costs, in particular transfer stamp duty and the bid-ask spread. These transaction costs reduced considerably over the full fifteen year period observed in

Chapter 4 which is consistent with the finding in that chapter that discount volatility declined over the period.

One of the main arguments in favour of the Lee *et al* (1991a) investor sentiment theory is that the wide variations in US discounts over time could only be due to individual investors (acting as noise traders) operating in the market. Yet we find no evidence that the proportion of shareholders who are individuals has any influence on discount volatility in the current UK study.

5.4 COMPARISON WITH RESULTS FOR US CLOSED-END FUNDS

There has been little empirical analysis of the cross-sectional variation in discount volatility for US closed-end funds, with the notable exception of Hoskins (1994).

Although discount volatility is calculated using weekly returns in Hoskins' US study, there is much similarity between his results and the results for UK trusts using monthly returns in this chapter. There are only two significant explanatory variables reported by Hoskins but they coincide with two of the significant explanatory variables (at the 0.5% level, two tail test) in the current study - trust share turnover ('trading velocity') and standard deviation of NAV return.

As discussed in 3.4.6, it is puzzling that total assets, which should be a reasonable indicator of the level of liquidity for fund shares, is not a significant explanatory variable in Hoskins' analysis. This might be due to a multicollinearity problem but unfortunately Hoskins does not give a cross-correlation matrix for his explanatory variables. In contrast, ln(market value) which should be a reasonable indicator of liquidity for trust shares, has the expected sign and is significant at the 0.5% level (two-tail test) in the current UK study. The final explanatory variable, percentage of underlying assets which are unquoted, is not included in the Hoskins study.

5.5 SUMMARY

In Chapter 5 we try to explain the cross-sectional variation in discount volatility for the UK investment trust sector and draw conclusions as to why discounts fluctuate so widely over time. The sample consists of 59 UK conventional investment trusts in continuous operation over the five years from 1 January 1992 to 31 December 1996. Discount volatility is calculated using monthly returns. Four explanatory variables are highly significant - trust share turnover, standard deviation of NAV return, ln(market value) and percentage of underlying assets which are unquoted. An adjusted R-square of 0.74 is obtained with these four explanatory variables. There is, however, a correlation coefficient of 0.58 between the first two variables and the Belsley condition index confirms that there is a multicollinearity problem.

The likely reasons for the significance of the four variables are as follows. *Trust share turnover* is related to the level of information hitting the market and information is the central driving force for share price movements. *Standard deviation of NAV return* proxies for both the ability and the need to hedge underlying net assets from the discount anomaly trader's perspective. *Percentage of underlying assets which are unquoted* is significant because valuations of unquoted assets tend to be historic which reduces the correlation between share price returns and NAV returns. *Ln(market value)* of the trust proxies for marketability (bid-ask spread), and the more marketable the trust shares, the narrower the discount trading range. The last two variables are both concerned with liquidity (of the underlying assets and the trust shares themselves respectively).

The results of this chapter suggest that the main driving forces for discount movements are, firstly, new information hitting trust share prices and, secondly, volatility of NAV returns. Discount arbitrage traders try to take advantage of discount anomalies but their activities are restricted by transaction costs and limited time horizons. There is no evidence that either individual investor sentiment or UK specific sentiment has any impact on discount volatility.

CHAPTER 6 - SENSITIVITY MEASURES FOR SPLIT CAPITAL INVESTMENT TRUSTS

6.1 INTRODUCTION

Many split capital trusts were created in the late 1980s or early 1990s, a period characterised by high levels of inflation and high interest rates by recent standards. The subsequent reduction in inflation and interest rates had a considerable impact on certain types of split securities due to their in-built gearing. This brought home the risks inherent in many split securities and highlighted the need for statistics which would measure the impact of changes in the underlying fundamental variables on different split securities.

Statistical measures of risk such as total risk, which are normally based on historical data, are useful tools for conventional trusts but are of little use for splits. The limited life of splits means that the risk profile of their securities change over time and analysts are often more concerned with sensitivity to the underlying fundamental variables. Furthermore, the traditional statistics used by analysts to assess the risks of split securities, which were described and discussed in 2.6.9, are very crude measures of risk and give little indication of price volatility.

It is possible to gain an insight into the risk of many split securities using 'sensitivity measures'. These show the proportionate change in the present value of a security caused by a small change in each of the underlying fundamental variables. In other words, they show how various split securities *should* behave in response to changes in the underlying fundamental variables. In practice, of course, split security prices will include a random element which may affect the actual response.

Split capital investment trusts are complex but, ultimately, they just divide up the returns from the underlying portfolio of investments in a particular way. This is similar, in principle, to issuing debt and equity capital. Sensitivity of the individual split securities with respect to a given fundamental variable (and weighted by their present values) should therefore 'add up' to the sensitivity of the underlying portfolio of investments with respect to that variable.

Hardly any academic research has been published on split capital investment trusts. There were a few papers published in the 1970s on US dual purpose funds, but little of that research has relevance to this thesis.

Simple valuation models are developed for traditional split securities in Section 6.2 and for quasi-split securities in Section 6.3. Estimation of the fundamental variables which determine the present values of split securities is then discussed in Section 6.4. Section 6.5 demonstrates that all the basic split securities consist of one or more of three types of component cash flow. This helps in the development of sensitivity measures for split securities which is covered in Sections 6.6, 6.7 and 6.8. The uses of sensitivity measures are discussed in Section 6.9 and the actual response of split securities to changes in the underlying fundamental variables observed in the market is tested in Section 6.10.

The theory is developed in discrete time. This is consistent with reality in that dividend payments are discrete. Investment trust analysts also generally employ 'rates' rather than 'forces' for the underlying fundamental variables. However, the mathematics is simplified if dividends are assumed to be paid continuously and if the theory is developed in continuous time rather than in discrete time. This approach is adopted in Adams (1999) which is included at end of this thesis.

6.2 TRADITIONAL SPLITS

As explained in 2.6.1, a basic traditional split has its ordinary share capital divided into two distinct categories - income shares and capital shares. Holders of income shares are entitled to all the distributed income and a predetermined capital repayment on liquidation. Holders of capital shares receive no income but are entitled to the remaining assets on liquidation after the income shares have been redeemed.

Capital shares are effectively European call options[1] on the underlying fund, with the predetermined capital repayment of income shares providing the exercise price of the

[1] They are like European calls on non dividend paying stock, but are unusual options in the sense that the underlying asset (portfolio of shares) can be changed by the fund managers before the expiry date. There are

options. The holders of income shares effectively own the underlying fund but are also writers of the call options.

By considering a simplified model for dual purpose funds in which dividends (and management fees) are proportional to NAV, Ingersoll (1976) develops pricing formulae for the capital and income shares of US dual purpose funds, the US equivalent of traditional split capital investment trusts. The model predicts share price fluctuations quite well and could be applied to UK traditional splits. However, Ingersoll uses the option pricing methods developed by Black-Scholes whereby the formulae are derived by calculating the price at which an option would have to stand in the market to allow a risk-free 'hedge' between the option and the underlying asset.[2] As a result, many of the fundamental variables (see Section 6.4) which are important for risk assessment purposes are not explicitly present in such formulae and so the 'sensitivities' derived from the Black-Scholes formula (such as delta, theta and kappa - see Hull, 1997) are of limited use from the point of view of risk assessment.[3] Practitioners cannot incorporate their own views about possible changes in the underlying fundamental variables which affect the value of split securities.

In this chapter we only consider trusts in which the capital shares are deep in-the-money as is normally the case (see Appendix 6A) so that the options have negligible time value and an approach based on option theory is not required for either the capital shares or the income shares.

Even before the introduction of a gilt strips market in 1997, risk-free spot rates[4] have been readily available from the gilt market, enabling cash flows from securities at different times to be valued at different appropriate discount rates (including risk premiums). This

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conflicts of interest between income and capital shareholders, and the managers. By deliberately changing the portfolio composition, the managers are able to change the allocation between different stakeholders.

[2] Estimation of the parameters in the Black-Scholes model applied to splits is reasonably straightforward. For example, volatility can be estimated from the standard deviation of the return on the trust's fund, which itself can be linked to the variances and covariances of the returns on the individual investments which comprise the fund.

[3] An example of the use of one of the Black-Scholes sensitivity measures, kappa, from the point of view of the managers of a traditional split trust would arise if the managers were thinking of disposing of certain investments and buying others as replacements. The change in volatility could be estimated prior to such action, and the consequent effect on the value of the capital shares deduced by calculating kappa.

[4] A spot rate is a rate paid when money is borrowed now to be repaid at a single date in the future.

information could be used in valuing split securities to make discount rates vary with the term of the dividend or capital payment and, of course, this would be an important consideration in valuing the capital share option if it were less deeply in-the-money. But in practice, redemption yields or present values based on fixed discount rates are used by investment analysts in assessing split capital securities, and this approach is convenient for calculating sensitivity measures in this chapter. We will also be working in real terms rather than in nominal terms so we will effectively assume that the spot rate curve for index-linked gilts is flat.

There may be other types of securities in the capital structure of traditional splits including zero dividend preference shares and warrants. We will not consider these more complicated capital structures further but the approach to risk analysis in this chapter may be applied to these more complicated structures provided that the capital share options (and warrants, if any) are deep in-the-money.

To simplify the discussion, we assume that the number of income shares is equal to the number of capital shares within the capital structure of the trust in question, as is often the case in practice. Inflation, real growth of dividends and real growth of the underlying fund are all assumed to be constant.

6.2.1 Valuation model for income shares

Income shares can be valued by discounting the estimated future dividends plus capital repayment. We assume for simplicity that dividends are paid annually and that the first dividend will be received in exactly one year from now.

Dividends from the equities in the underlying portfolio are paid out of profits which in the medium term tend to increase as the general level of prices increases plus any real growth. The absolute level of dividend growth of the income shares should thus depend on the level of future inflation, and in valuing the dividends from the income shares it is therefore helpful to work in real terms rather than in nominal terms.

The present value of an income share is given by:

$$V_0 = \frac{D_1}{1+j}\sum_{k=0}^{m-1}\left(\frac{1+g}{1+j}\right)^k + \frac{R}{(1+f)^m(1+j)^m}$$

where D_1 is the estimated dividend per income share payable one year from now (in current money terms[5]).

g is the estimated real dividend growth rate per annum.

j is the real discount rate per annum, assumed to apply to all cash flows.

f is the estimated rate of inflation per annum.

m is the period (in years) before the income shares are redeemed.

R is the redemption amount for each income share after m years.

$$\Rightarrow V_0 = D_1\left\{\frac{1-\left(\frac{1+g}{1+j}\right)^m}{j-g}\right\} + \frac{R}{(1+f)^m(1+j)^m} \tag{6.1}$$

6.2.2 Valuation model for capital shares

Capital shares receive no income so their return depends entirely on the growth of underlying assets up to the wind-up date. We have assumed that there is little doubt that there will be sufficient assets at the wind-up date to pay the entitlement of the income shares, so the present value of a capital share is:

$$V_0 = \frac{A_0(1+g^*)^m}{(1+j)^m} - \frac{R}{(1+f)^m(1+j)^m} \tag{6.2}$$

where A_0 is the value of the fund at time 0 per capital share.

g^* is the estimated real growth rate per annum of the fund.

R is the redemption amount of the income shares per capital share[6].

j, f and m are defined as in 6.2.1

[5] In other words, D_1 is equal to the historic dividend which has just been paid increased in line with the expected real growth of the dividend over the next year.

[6] This is the same as the redemption amount per income share because we have assumed that the number of capital shares is equal to the number of income shares.

There is freedom to choose g^* to be different from g, unlike in the Ingersoll (1976) model for dual purpose funds where dividend payments are proportional to the value of the underlying fund so that g^* is set equal to g.

6.3 QUASI-SPLITS

As explained in 2.6.2, quasi-splits always have zero dividend preference shares (ZDPs) in issue but there is only one class of ordinary share capital, namely income & residual capital shares. When such a trust is wound up, ZDPs are repaid first.

The income & residual capital shareholders effectively own the underlying fund less the discounted redemption amount of the ZDPs, but also hold a European put option[7] on the underlying fund, with the exercise price equal to the redemption amount of the ZDPs. Holders of the ZDPs are writers of the put options, with the exercise price of the put equal to the redemption amount of the ZDPs; the fair value of the ZDPs is the discounted redemption amount less the value of the put option which they have written. Again, we only consider trusts in which the put option is of negligible value as is often the case (see Appendix 6B) so that option valuation models are not required. We also assume a flat spot rate curve for gilts as in Section 6.2. Quasi-splits with bank borrowing or with additional types of securities, such as stepped preference shares or warrants, are not considered.

To simplify the discussion, we assume that the number of ZDP shares is equal to the number of income & residual capital shares within the capital structure of the trust in question. Inflation, real growth of dividends and real growth of the underlying fund are again all assumed to be constant.

6.3.1 Valuation model for zero dividend preference shares

ZDPs pay no income but pay a fixed capital sum on a fixed date. The present value of a ZDP share is therefore given by:

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[7] Again, they are unusual options in the sense that the underlying asset (portfolio of shares) can be changed by the fund managers before the expiry date.

$$V_0 = \frac{R}{(1+f)^m (1+j)^m} \tag{6.3}$$

where R is the redemption amount of the ZDPs.

m is the period (in years) before the ZDPs are redeemed.

j is the real discount rate per annum, assumed to apply to all cash flows.

f is the estimated rate of inflation per annum.

6.3.2 Valuation model for income & residual capital shares

Income & residual capital shares offer high income plus all the remaining assets of a quasi-split trust at the wind-up date, after the ZDPs have received their capital entitlement. Valuing these shares requires an estimate of both income growth and growth of the underlying assets up to the wind-up date. We assume that dividends are paid annually and that the first dividend will be paid in exactly one year from now. The same discount rate is used for income and capital.

The present value of an income & residual capital share is then given by:

$$V_0 = \frac{D_1}{1+j} \sum_{k=0}^{m-1} \left(\frac{1+g}{1+j} \right)^k + \frac{A_0 (1+g^*)^m}{(1+j)^m} - \frac{R}{(1+f)^m (1+j)^m}$$

where D_1 is the estimated dividend per income & residual capital share payable one year from now (in current money terms).

g is the estimated real dividend growth rate per annum.

A_0 is the value of the underlying fund at time 0 per income & residual capital share.

g^* is the estimated real growth rate per annum of the fund.

R is the redemption amount of the ZDPs per income & residual capital share[8].

j, f and m are defined as in 6.3.1.

[8] This is the same as the redemption amount per ZDP share because we have assumed that the number of income & residual capital shares is equal to the number of ZDP shares.

Hence $$V_0 = D_1 \left\{ \frac{1-\left(\frac{1+g}{1+j}\right)^m}{j-g} \right\} + \frac{A_0(1+g^*)^m}{(1+j)^m} - \frac{R}{(1+f)^m(1+j)^m} \tag{6.4}$$

6.4 ESTIMATING THE FUNDAMENTAL VARIABLES

6.4.1 Real growth rate of dividends (*g*)

Real dividend growth of the income shares (traditional split) or income & residual capital shares (quasi-split) are directly related to the real growth of dividends from the underlying fund which will normally be invested in UK equities. Consideration of real dividend growth from the UK equity market may therefore provide a useful starting point in the estimation procedure. It must be remembered, however, that past dividend growth may have been financed partly by a general reduction in dividend cover, which in turn may be influenced by factors such as taxation and dividend controls. Real dividend growth from the UK equity market has been 2.1% p.a. over the last fifty years.[9]

6.4.2 Real growth rate of the underlying fund (*g**)

This estimate is clearly of crucial importance in the valuation of capital shares (traditional splits) and income & residual capital shares (quasi-splits). Real growth of an appropriate index may provide a useful tentative starting point in the estimation procedure. However, growth in capital values will partly depend on changing levels of retentions and yields. The UK equity market has given real growth in capital values of 1.7% p.a. over the last fifty years.[10]

6.4.3 Future rate of inflation (*f*)

The market's estimate of the future rate of inflation may be derived from the gilt market. A first order estimate is simply the redemption yield on conventional gilts less the real

[9] Based on the notional dividend of the FT-SE Actuaries All Share Index and, before it commenced in 1962, the BZW Equity Index. For the future, ACT/tax credit changes may lead to an increase in retentions, and the current emphasis on 'buy backs' as an alternative to dividends for many companies requires careful consideration.

[10] Based on the FT-SE Actuaries All Share Index and, before it commenced in 1962, the BZW Equity Price Index. As the latter index excludes small companies, the stated historic growth rate is a slight underestimate.

redemption yield[11] on index-linked gilts of similar duration.[12] An adjustment can then be made to reflect the different risk premiums on the different types of instrument and the various technical inaccuracies[13] which arise from this measure (see Deacon and Derry, 1994). Investors can make further adjustments in line with their own views of the future course of the economy.

6.4.4 Real discount rate (*j*)

The real discount rate is the real rate of return required by investors. There are a number of approaches that can be adopted in determining the real discount rate for a particular split security, depending on the purpose of the analysis.[14]

If the purpose is to assess whether the market price of a particular split security is cheap or dear using the formulae in Sections 6.2 or 6.3, the real discount rate for a particular split security could be estimated from similar securities, given their market prices. Alternatively, the real discount rate could be estimated using the investor's own judgement. This will involve choosing a required risk premium above the risk-free real rate of return (perhaps obtained from the market for index-linked gilts). For traditional splits, a lower risk premium would be required for income shares compared with capital shares as they rank before capital shares and receive all the income from the underlying portfolio. Similarly, for quasi-splits, a lower risk premium would be required for ZDPs as compared with income & residual capital shares. For income shares and ZDPs, higher risk premiums would be associated with less negative hurdle rates and lower cover. If the market is efficient and ignoring taxation effects, the weighted average of the risk premiums for the individual securities of a particular split trust should equal the risk premium appropriate to the underlying equity portfolio.[15]

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[11] The real redemption yield itself requires an inflation assumption.

[12] It has been argued (Debt Management Office, 1999) that the real redemption yields on long-dated index-linked gilts are artificially depressed at present as a result of the minimum funding requirement (MFR) valuation rules for pension funds. The increased demand which may be arising as a result of the application of these rules is happening at a time of reduced supply due to a reduction in government deficits.

[13] These include the inflation risk premium for conventional gilts and the inflation lag for index-linked gilts.

[14] It may be argued that different types of cash flow (e.g. bond-type income as against equity-type income) from a particular split security should be valued using different real discount rates. This complication will not be considered although refinements which deal with this point could be introduced into the models we derive.

[15] There are different views on the appropriate risk premium for the UK equity market. Dimson (1993) suggests a figure of 7% p.a. based on the historic return on shares as compared with the total returns on fixed-interest stocks. The Monopolies and Mergers Commission (1995) proposes a range of between 3.5% and

For other purposes, it may be appropriate to derive the real discount rate from the market. This involves replacing the present value (V_0) in the formulae in Sections 6.2 or 6.3 by the actual market price of the security, substituting estimated values for the other fundamental variables (g, g^* and f) and then solving for the real discount rate.

6.5 THE COMPONENT CASH FLOWS OF THE BASIC SPLIT SECURITIES

Examination of equations (1), (2), (3) and (4) shows that all the basic split securities considered in this paper can be thought of as consisting of one or more of the three component cash flows (positive or negative), with present values W_1, W_2 and W_3, given in Table 6.1.

Table 6.1: Component cash flows of the basic split securities

Cash flow	Present value
Stream of dividends for m years	$W_1 = D_1 \left\{ \dfrac{1 - \left(\dfrac{1+g}{1+j} \right)^m}{j-g} \right\}$
Nominal amount R after m years	$W_2 = \dfrac{R}{(1+r)^m}$ where $(1+r)^m = (1+f)^m (1+j)^m$
Value of fund after m years (per share)	$W_3 = \dfrac{A_0 (1+g^*)^m}{(1+j)^m}$

Thus,

PV of income shares = $W_1 + W_2$

PV of capital shares = $W_3 - W_2$

PV of ZDPs = W_2

PV of income & residual capital shares = $W_1 + W_3 - W_2$

4.5% p.a. Wilkie (1995) suggests a premium of 2% to 5% p.a. above the yield on index-linked stocks over a reasonably short period (less than five years) and about 1% p.a. lower than this over a longer period.

It is useful to treat split securities as consisting of these component cash flows when deriving their sensitivity measures in the following sections.

6.6 SENSITIVITY MEASURES

Sensitivity measures for equity investments were developed by Adams and Booth (1995 - paper included at end of thesis). They are an extension to the concept of duration which was developed by Macaulay (1938) and is commonly used in fixed-interest bond markets. Duration is defined as the weighted average of the times of receipts from an investment, where the weights are equal to the present value of the payments. Duration is effectively a mathematical measure of the sensitivity of an investment value to changes in the nominal discount rate in the perfect-certainty discounted cash flow (DCF) model.

Although duration has generally been applied to the risk analysis of bonds, the concept has also been applied to the risk analysis of equities (Boquist *et al* (1975), Casabona *et al* (1984)) using a constant dividend growth model expressed in nominal terms. A more recent development by Leibowitz *et al* (1989) is the recognition that inflation, which is a major component of the nominal discount rate, is also a major component of the nominal dividend growth rate. As a result, the sensitivity of equity prices to nominal interest rates is, in practice, far less than that implied by the traditional equity duration model.

Sensitivity measures show how the present value of expected future cash flows will vary as the fundamental variables which determine the present value vary. The analysis is valid for sensitivities to factors other than nominal discount rate. Sensitivity measures can be calculated with respect to all the underlying fundamental variables, whether they be real variables or nominal variables. For split capital investment trusts, these fundamental variables will include the estimated rate of inflation, the real discount rate, and the estimated real growth rates of both dividends and the underlying fund.

In this thesis, we define sensitivity to variable k as:

$$S_k = \frac{\partial V_0}{\partial k} \cdot \frac{1}{V_0}$$

Sensitivity to variable k can be thought of as the percentage change in the present value of the investment per one percentage point change in variable k, for small changes in variable k.

Sensitivity measures are clearly of use in comparing the risk of different split securities. But the following should be borne in mind:

(i) The importance of a given variable in influencing the present value of a particular security over time depends not only on the sensitivity measure with respect to that variable but also on the volatility of that variable itself over time. Thus, although the sensitivity measure for a particular security with respect to variable k may be relatively low, if variable k is extremely volatile over time, it may still have an important influence on changes in the present value of that security over time.

(ii) The sensitivity measures are all partial, and do not allow for the fact that the various determinants may be interdependent. For example, real discount rates may tend to fall as expected inflation rises.

6.6.1 Sensitivity measures and component cash flows

Suppose that the cash flow from a security may be split into n different component cash flows. If the present value of the ith component cash flow is B_i ,

$$V_0 = \sum_{i=1}^{n} B_i$$

Then sensitivity with respect to a fundamental variable k is:

$$S_k = \frac{\partial V_0}{\partial k} \cdot \frac{1}{V_0}$$

$$= \sum_{i=1}^{n} \frac{\partial B_i}{\partial k} \cdot \frac{1}{B_i} \cdot \frac{B_i}{V_0}$$

$$\text{or } S_k = \sum_{i=1}^{n} S_k^{B_i} \cdot \frac{B_i}{V_0} \tag{6.5}$$

where $S_k^{B_i}$ is the sensitivity to variable k of the ith component cash flow. Thus:

> *Sensitivity is equal to the weighted average of the sensitivities of the component cash flows where the weights are equal to the present values of the component cash flows.*

This is analagous to, but more general than, the result that the duration of a portfolio of investments is the weighted average of the durations of the individual investments, where the weights are equal to the present values of the individual investments (see, for example, McCutcheon and Scott, 1991)[16].

6.6.2 Sensitivities of the component cash flows of splits

The sensitivities of the component cash flows of split securities are given in Table 6.2.

[16] In continuous time (see Adams, 1999) the sensitivity measures can be interpreted as: the weighted average of the durations of the component cash flows which are sensitive to that particular fundamental variable. The weights are equal to the present values (positive or negative) of the component cash flows.

Table 6.2: Sensitivities of the component cash flows of split securities

	Real discount rate sensitivity	Real growth rate sensitivity	Inflation sensitivity
Stream of dividends for m years	$\frac{-1}{j-g}+\frac{m}{1+j}\cdot\frac{\left(\frac{1+g}{1+j}\right)^m}{1-\left(\frac{1+g}{1+j}\right)^m}$	$\frac{1}{j-g}-\frac{m}{1+g}\cdot\frac{\left(\frac{1+g}{1+j}\right)^m}{1-\left(\frac{1+g}{1+j}\right)^m}$	0
Nominal amount after m years	$\frac{-m}{1+j}$	0	$\frac{-m}{1+f}$
Value of fund after m years	$\frac{-m}{1+j}$	$\frac{m}{1+g^*}$	0

Henceforth, we will use the following notation for the cells of Table 6.2.

S_k^1 = sensitivity of a stream of dividends for m years with respect to variable k.

S_k^2 = sensitivity of a nominal amount payable after m years with respect to variable k.

S_k^3 = sensitivity of the value of the underlying fund after m years with respect to variable k.

The expressions in Table 6.2 are useful for the derivation of sensitivity measures for split securities to which we now turn.

6.7 SENSITIVITY MEASURES FOR TRADITIONAL SPLITS

6.7.1 Income shares

Present value = $W_1 + W_2$

Hence using equation (6.5) we obtain:

$$S_k = \frac{W_1 S_k^1 + W_2 S_k^2}{W_1 + W_2} \qquad (k = j, g \text{ or } f) \tag{6.6}$$

Thus, using Tables 6.1 and 6.2, sensitivity to real discount rate is given by:

$$S_j = \frac{-D_1\left\{\dfrac{-m(1+g)^m}{(1+j)^{m+1}} \cdot \dfrac{1}{j-g} + \dfrac{1-\left(\dfrac{1+g}{1+j}\right)^m}{(j-g)^2}\right\} - \dfrac{mR}{(1+f)^m} \cdot \dfrac{1}{(1+j)^{m+1}}}{D_1\left\{\dfrac{1-\left(\dfrac{1+g}{1+j}\right)^m}{j-g}\right\} + \dfrac{R}{(1+f)^m(1+j)^m}}$$

Sensitivity to real growth rate of dividends is given by:

$$S_g = \frac{D_1\left\{\dfrac{-m(1+g)^{m-1}}{(1+j)^m} \cdot \dfrac{1}{j-g} + \dfrac{1-\left(\dfrac{1+g}{1+j}\right)^m}{(j-g)^2}\right\}}{D_1\left\{\dfrac{1-\left(\dfrac{1+g}{1+j}\right)^m}{j-g}\right\} + \dfrac{R}{(1+f)^m(1+j)^m}}$$

Sensitivity to rate of inflation is given by:

$$S_f = \frac{\dfrac{-mR}{(1+f)^{m+1}(1+j)^m}}{D_1\left\{\dfrac{1-\left(\dfrac{1+g}{1+j}\right)^m}{j-g}\right\} + \dfrac{R}{(1+f)^m(1+j)^m}}$$

These sensitivity measures may be evaluated for a particular issue of income shares by choosing the appropriate values for the underlying variables (see Section 6.4). If the real discount rate is obtained from the market then the denominator of the right hand side of each of the above sensitivity equations is simply equal to the market price of the income shares.

6.7.2 Capital shares

Present value = W_3 -W_2

Hence using equation (6.5) we obtain:

$$S_k = \frac{W_3 S_k^3 - W_2 S_k^2}{W_3 - W_2} \qquad (k = j, g^* \text{ or } f) \tag{6.7}$$

Thus, using Tables 6.1 and 6.2, sensitivity to real discount rate is given by:

$$S_j = \frac{-m}{1+j}$$

Sensitivity to real growth rate of the underlying fund is given by:

$$S_{g^*} = \frac{mA_0(1+g^*)^{m-1}}{A_0(1+g^*)^m - \dfrac{R}{(1+f)^m}}$$

Sensitivity to rate of inflation is given by:

$$S_f = \frac{mR}{A_0(1+g^*)^m(1+f)^{m+1} - R(1+f)}$$

Again, these sensitivity measures may be evaluated for a particular issue of capital shares by choosing appropriate values for the underlying variables.

6.7.3 Example

A traditional split trust is invested entirely in a diversified portfolio of UK equities and has equal numbers of income shares and capital shares in its capital structure. Details of the trust are as follows:

Estimated dividend per income share, in current money terms (D_1) = 10p

Redemption amount for each income share (R) = 50p

Value of underlying fund at time 0 per capital share (A_0) = 200p

Period before income shares are repaid (m) = 10 years

The income shares stand at a price of $93^1/_4$ p in the market and the capital shares stand at a price 81p. The 10 year par yield for conventional gilts is 8% and the real redemption yield on a 10 year index-linked gilt is 4%.[17]

We first estimate the underlying fundamental variables.

The estimated rate of inflation (f) is approximately 4% p.a. This is obtained from the difference between the par yield on conventional gilts and the par real yield on index-linked gilts. Based on historic growth figures for the UK equity market (see Section 6.4) we estimate real dividend growth (g) of 2% p.a. and real growth of the underlying fund (g^*) of 2% p.a.

Given these estimates and setting V_0 equal to $93^1/_4$ in equation (6.1) we can solve for the real discount rate (j) for the income shares and obtain a value of 7% p.a. Similarly, setting

[17] Assumes inflation at 4%.

V_0 equal to 81 in equation (6.2) we obtain a real discount rate (j) for the capital shares of 10% p.a.

We then obtain the following sensitivity measures for the income shares and the capital shares using equations (6.6) and (6.7).

	Real discount rate sensitivity	Real growth rate sensitivity	Inflation sensitivity
Income shares	-5.62	+3.28	-1.77
Capital shares	-9.09	+11.38	+1.55

What does a figure of -5.62 for real discount rate sensitivity of the income shares mean? It simply means that there is a fall of 5.62% in the present value of the income shares per one percentage point rise in the real discount rate, for small changes in the real discount rate. The figure is more exact the smaller the change in real discount rate. Similarly, the real growth rate sensitivity of +3.28 for the income shares indicates that there is a rise of 3.28% in the present value of income shares per one percentage rise in the estimated real growth rate of dividends, for small changes in the real growth rate of the dividends. Other figures in the above table are to be interpreted in a similar way.

The figure of +11.38 for real growth rate sensitivity sums up in a single figure the sensitivity of capital shares to changes in the real growth rate of the underlying fund. It shows that there is a rise of 11.38% in the present value of capital shares per one percentage point rise in the estimated real growth rate of the underlying fund, for small changes in the estimated real growth rate of the fund. Section 2.6.9 described how analysts get an indication of the sensitivity of the capital shareholders' returns to the growth rate of the underlying fund by calculating the gross redemption yield for different assumed growth rates for the underlying fund. In comparing the risk of different split securities, this involves comparing a range of figures with another range of figures. Clearly it is more useful for many purposes to compare the sensitivity of different capital shares (and other split securities) to inflation and separately to real growth of the underlying fund using a single statistic in each case.

The income shares are less real discount rate sensitive than the capital shares because they have shorter duration. The income shares are less real growth rate sensitive than the capital shares because part of the return from the income shares (the redemption amount) is fixed in money terms. Income shares have negative inflation sensitivity because the redemption payment is a fixed nominal amount. Capital shares, on the other hand, have positive inflation sensitivity because the real value of the amount deducted from the fund and paid to income shareholders on wind-up is reduced by an increase in expected inflation.

It is of interest to see how the sensitivity measures vary for different values of the period to wind-up (m) rather than fixing $m = 10$.

Figures 6.1, 6.2 and 6.3 show how the sensitivity measures for the income shares change as the period to wind-up changes.

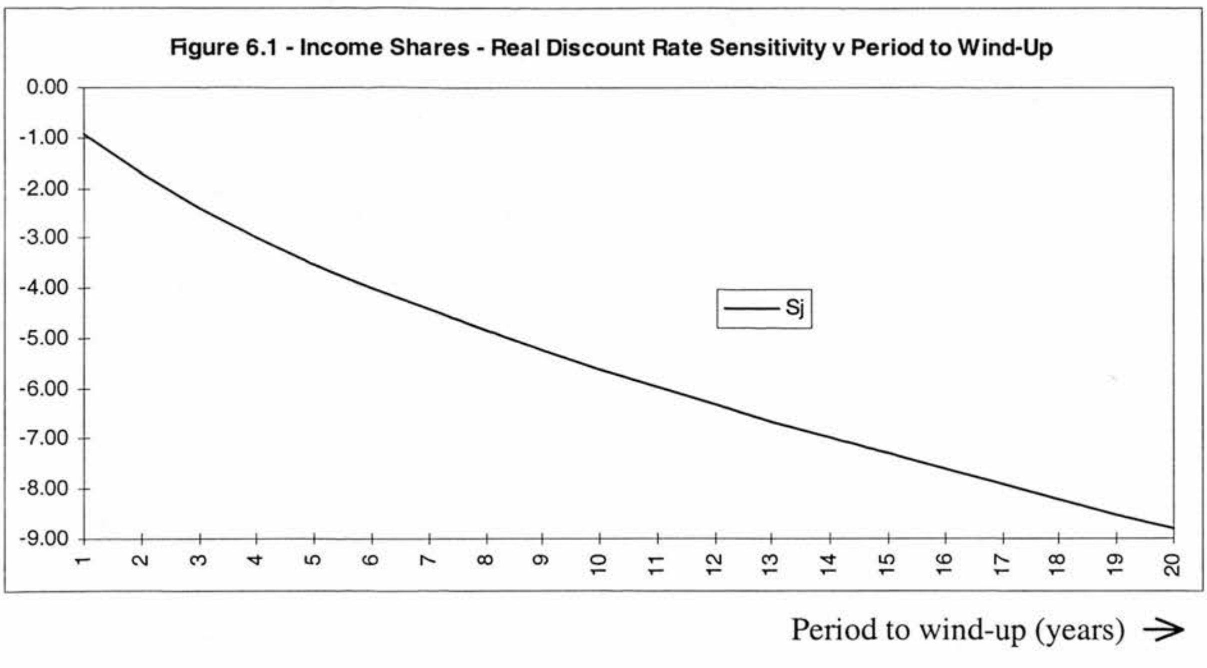


Figure 6.2 - Income Shares - Real Growth Rate Sensitivity v Period to Wind-Up

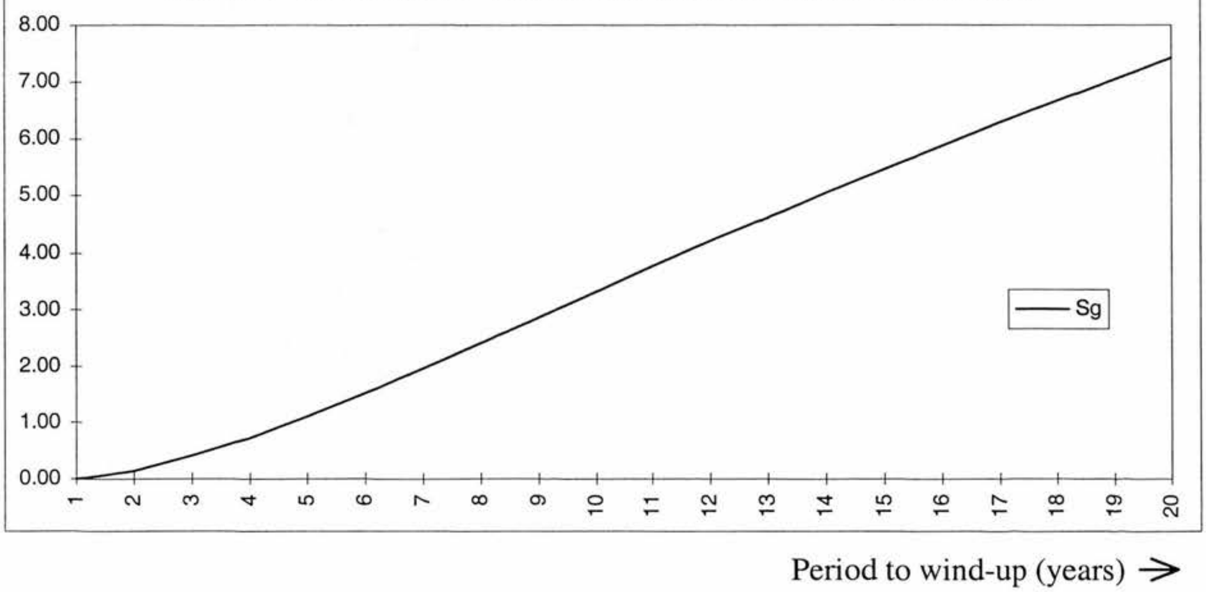
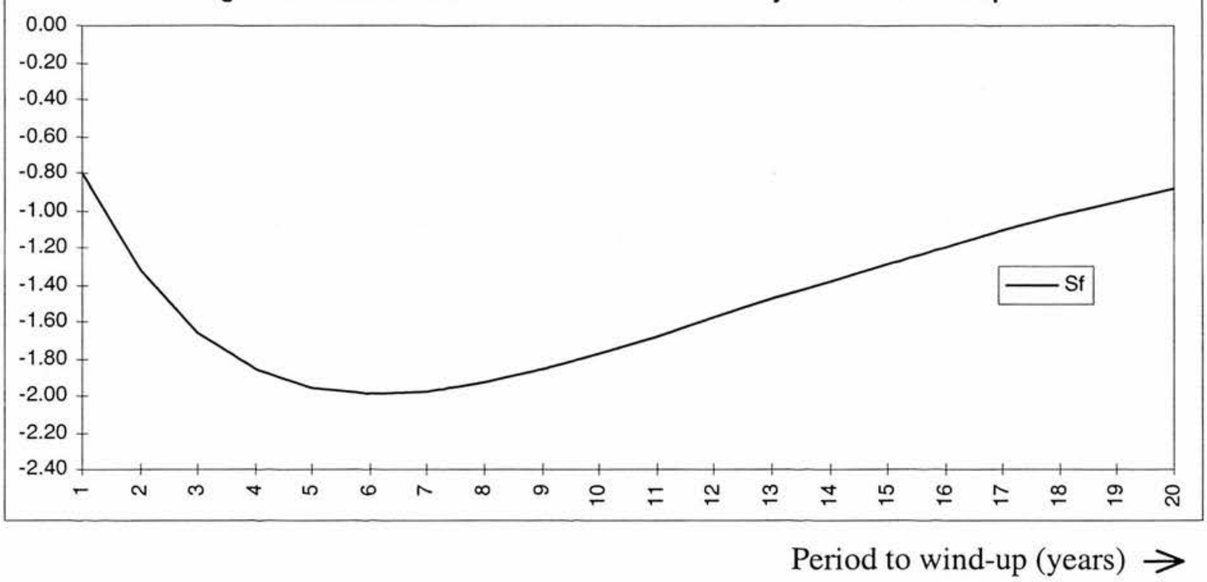


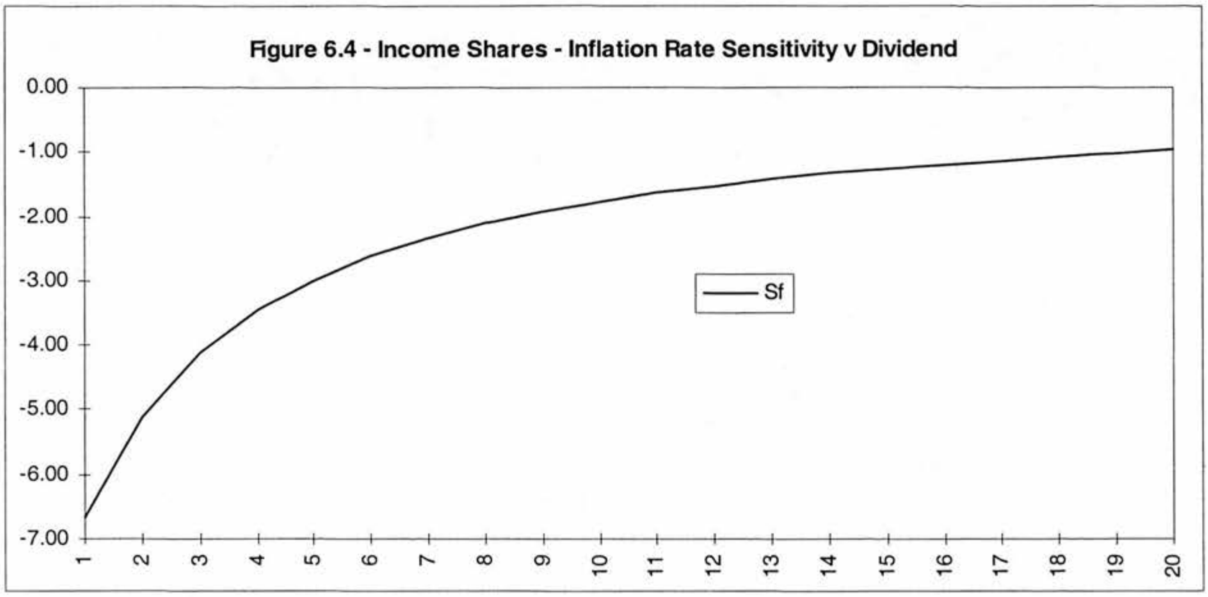
Figure 6.3 - Income Shares - Inflation Rate Sensitivity v Period to Wind-Up



As expected, Figures 6.1 and 6.2 show that the income shares are more real discount rate sensitive and more real growth rate (of dividends) sensitive as the period to wind-up is increased. Examination of the y-axis scale of Figure 6.3 shows that the income shares are not very sensitive to inflation expectations for all the values of *m*. However, inflation rate sensitivity becomes more important if the dividend (D_I) is lower in relation to the redemption amount (R). For example, if the redemption amount is held constant at 50p say, then we obtain:

$$S_f = -165.10 / (7.61\ D_I + 17.17)$$

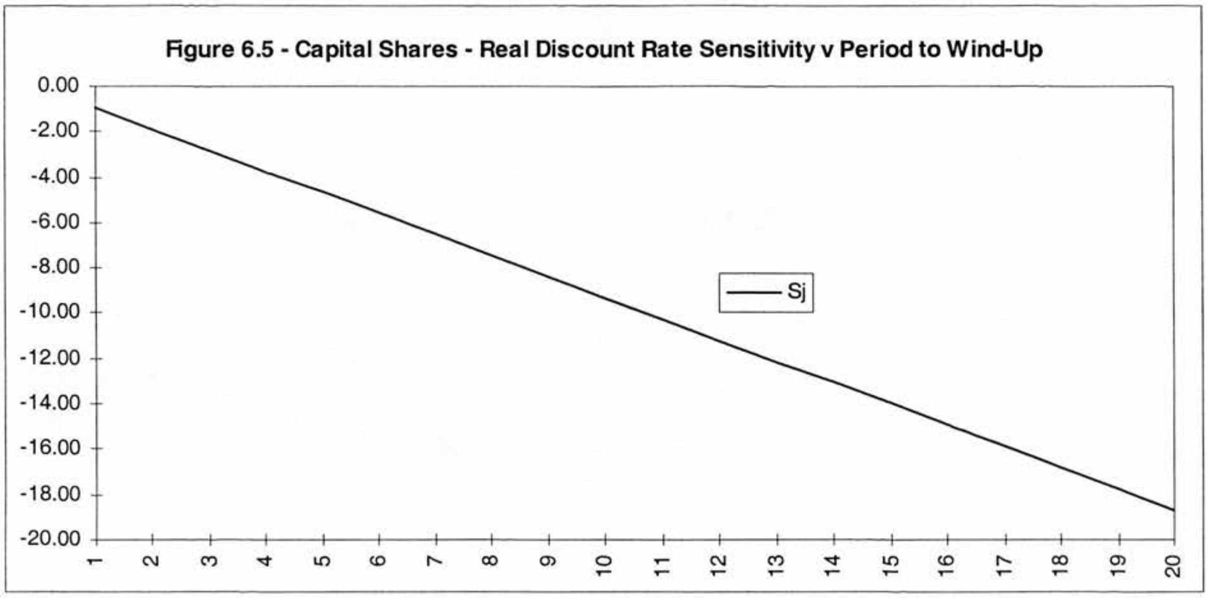
This is illustrated in Figure 6.4.



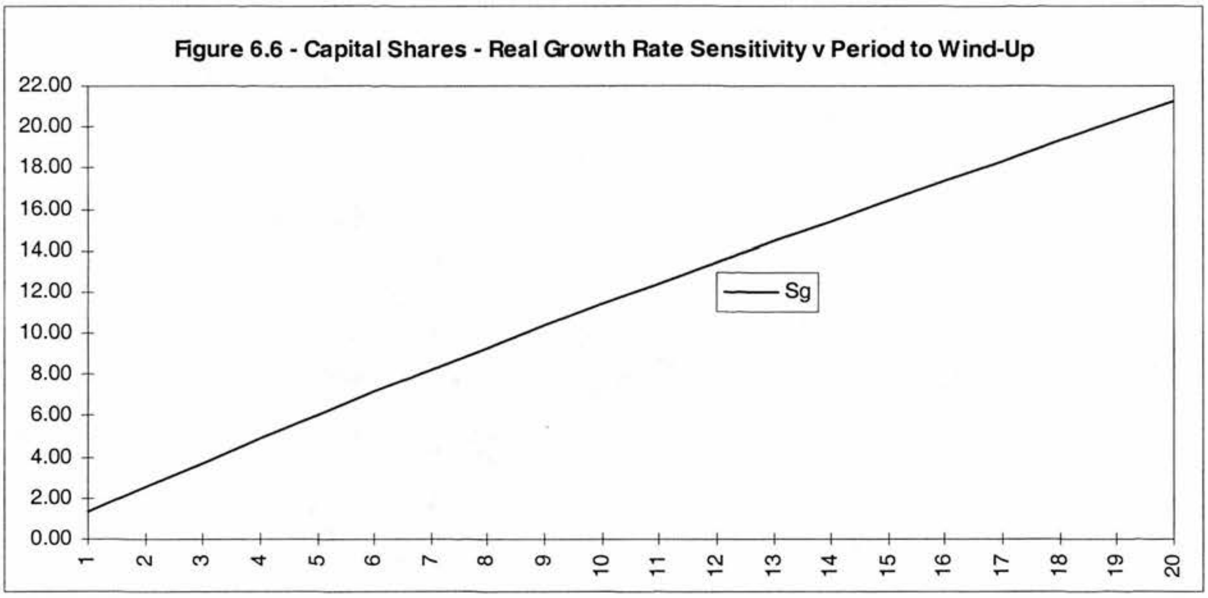
Dividend per income share (D_I) →

Figures 6.5, 6.6 and 6.7 show how the sensitivity measures for the capital shares change as the period to wind-up changes.

Figures 6.5 and 6.6 show that the capital shares are more real discount rate sensitive and more real growth rate (of the underlying fund) sensitive as the period to wind-up is increased.



Period to wind-up (years) →

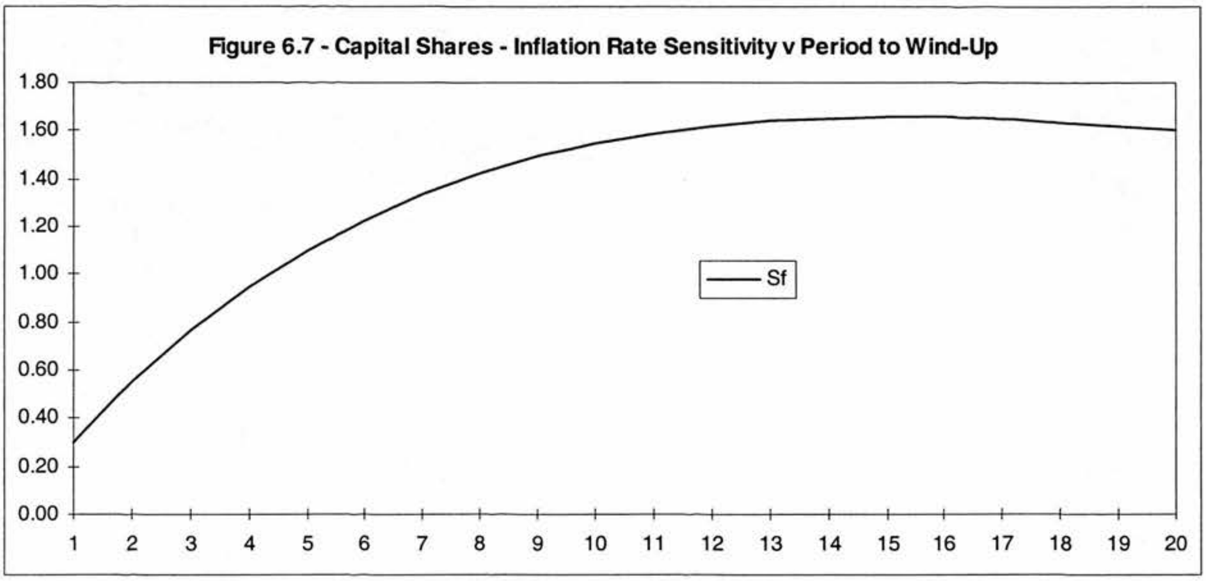


Period to wind-up (years) →

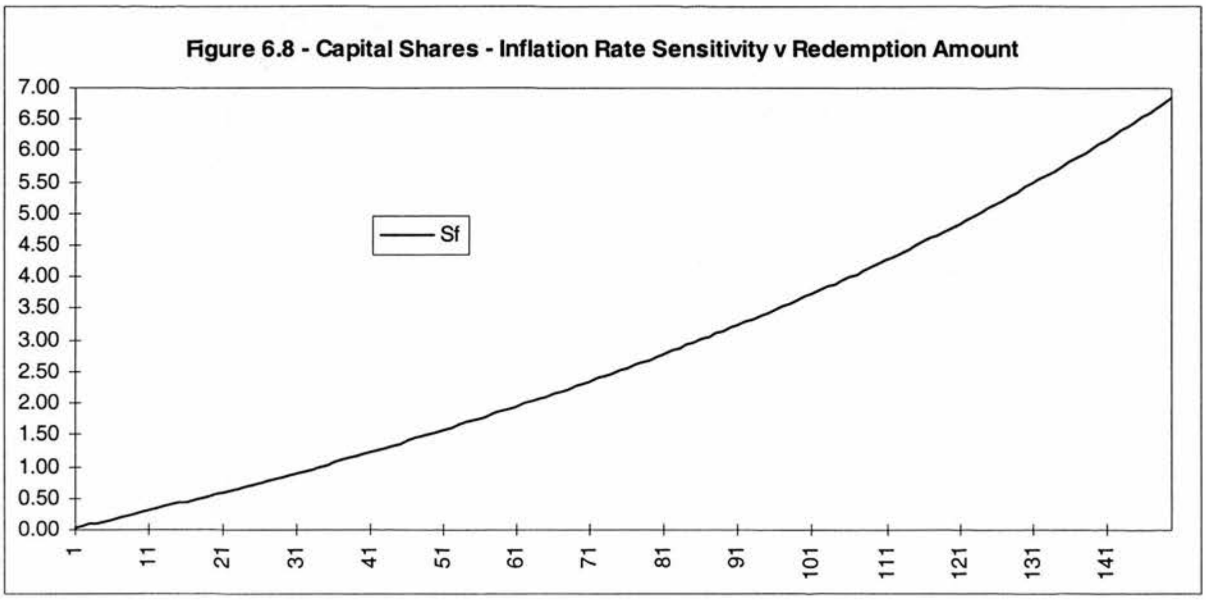
Examination of the y-axis scale of Figure 6.7 shows that the capital shares are not very sensitive to inflation expectations for all values of m. However, inflation rate sensitivity becomes more important if the redemption amount (R) of the income shares is higher in relation to the value of the underlying fund per share (A_0). In other words, the higher the level of capital share gearing, the higher the inflation rate sensitivity. For example, if the value of the underlying fund per share at time 0 is held constant at 200p say, then we obtain:

$$S_f = 10R / (375.32 - 1.04R)$$

This is illustrated in Figure 6.8.



Period to wind-up (years) →



Redemption amount of income share (R) →

6.8 SENSITIVITY MEASURES FOR QUASI-SPLITS

6.8.1 Zero dividend preference shares

Present value = W_2

Hence $\quad S_k = S_k^2 \qquad (k = j, g, f) \qquad (6.8)$

Thus, using Table 6.2, sensitivity to real discount rate is given by:

$$S_j = \frac{-m}{1+j}$$

and sensitivity to estimated rate of inflation is given by:

$$S_f = \frac{-m}{1+f}$$

6.8.2 Income & residual capital shares

Present value = $W_1 + W_3 - W_2$

This formula involves both the estimated real growth rate of dividends (g) and the estimated real growth rate of the underlying fund (g^*). These variables may not be independent. It is within the power of the trust managers to increase g, but in doing so they may reduce g^*. To simplify the discussion, however, we will consider only sensitivity to a single growth rate by setting $g^* = g$. Broadly speaking, the assumption here is that the dividend yield of the fund remains constant. Hence using equation (6.5):

$$S_k = \frac{W_1 S_k^1 + W_3 S_k^3 - W_2 S_k^2}{W_1 + W_3 - W_2} \qquad (k = j,\ g,\ f) \qquad (6.9)$$

Thus, using Tables 6.1 and 6.2, sensitivity to real discount rate is given by:

$$S_j = \frac{-D_1\left\{\frac{-m(1+g)^m}{(1+j)^{m+1}}\cdot\frac{1}{j-g}+\frac{1-\left(\frac{1+g}{1+j}\right)^m}{(j-g)^2}\right\}-\frac{mA_0(1+g)^m}{(1+j)^{m+1}}+\frac{mR}{(1+f)^m(1+j)^{m+1}}}{D_1\left\{\frac{1-\left(\frac{1+g}{1+j}\right)^m}{j-g}\right\}+\frac{A_0(1+g)^m}{(1+j)^m}-\frac{R}{(1+f)^{m(}(1+j)^m}}$$

Sensitivity to real growth rate is given by:

$$S_g = \frac{D_1\left\{\frac{-m(1+g)^{m-1}}{(1+j)^m(j-g)}+\frac{1-\left(\frac{1+g}{1+j}\right)^m}{(j-g)^2}\right\}+\frac{mA_0(1+g)^{m-1}}{(1+j)^m}}{D_1\left\{\frac{1-\left(\frac{1+g}{1+j}\right)^m}{j-g}\right\}+\frac{A_0(1+g)^m}{(1+j)^m}-\frac{R}{(1+f)^m(1+j)^m}}$$

Sensitivity to estimated rate of inflation is given by:

$$S_f = \frac{\frac{mR}{(1+f)^{m+1}(1+j)^m}}{D_1\left\{\frac{1-\left(\frac{1+g}{1+j}\right)^m}{j-g}\right\}+\frac{A_0(1+g)^m}{(1+j)^m}-\frac{R}{(1+f)^m(1+j)^m}}$$

If the real discount rate is obtained from the market then the denominator of the right hand side of each of the above sensitivity equations is simply equal to the market price of the income & residual capital shares.

6.8.3 Example

A quasi-split trust is invested entirely in a diversified portfolio of UK equities and has equal numbers of ZDP shares and income & residual capital shares in its capital structure. Details of the trust are as follows:

> Estimated dividend per income & residual capital share, in current money terms (D_1) = 10p
>
> Redemption amount of a ZDP share (R) = 50p
>
> Value of underlying fund at time 0 per income & residual capital share (A_0) = 200p
>
> Period before the ZDP shares are redeemed (m) = 10 years

The ZDPs stand at a price of 17 p in the market and the income & residual capital shares stand at a price 164p. The 10 year par yield for conventional gilts is 8% and the real redemption yield on a 10 year index-linked gilt is 4%.[18]

We first estimate the underlying fundamental variables.

The estimated rate of inflation (f) is approximately 4% p.a. This is obtained from the difference between the par yield on conventional gilts and the par real yield on index-linked gilts. Based on historic growth figures for the UK equity market (see Section 6.4) we estimate real dividend growth (g) of 2% p.a. and real growth of the underlying fund (g^*) of 2% p.a.

Given these estimates and setting V_0 equal to 17 in equation (6.3) we can solve for the real discount rate (j) for the ZDPs and obtain a value of 7.1% p.a. Similarly, setting V_0 equal to 164 in equation (6.4) we obtain a real discount rate (j) for the income and residual capital shares of 8.5% p.a.

We then obtain the following sensitivity measures using equations (6.8) and (6.9).

[18] Assumes inflation at 4%.

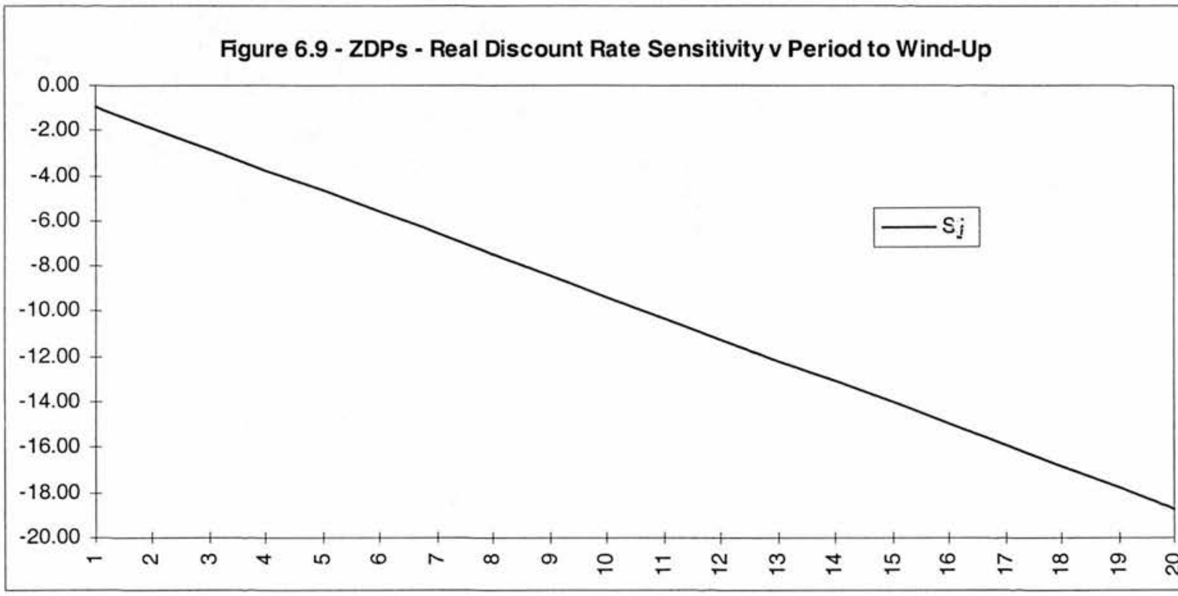
	Real discount rate sensitivity	Real growth rate sensitivity	Inflation sensitivity
ZDPs	-9.34	0	-9.62
Income & residual capital shares	-7.22	+8.15	+0.88

A figure of -9.34 for the real discount rate sensitivity of the ZDPs means that there is a fall of 9.34% in the present value of the ZDPs per one percentage point rise in the real discount rate, for small changes in the real discount rate. The figure is more exact the smaller the change in real discount rate. Other figures in the above table are to be interpreted in a similar way.

The figure of +8.15 for real growth rate sensitivity sums up in a single figure the sensitivity of income & residual capital shares to changes in the real growth rate of the underlying fund. It shows that there is a rise of 8.15% in the present value of the income & residual capital shares per one percentage point rise in the estimated real growth rate of the underlying fund, for small changes in the estimated real growth rate of the fund. As mentioned in 2.6.9, analysts get an indication of the sensitivity of the returns from these shares to the growth rate of the underlying fund by calculating the gross redemption yield for different assumed growth rates for the underlying fund. The approach is similar to that for capital shares. Again, it is more useful for many purposes to compare the sensitivity of different securities using a single statistic rather than try to compare a range of figures with another range of figures.

The ZDPs are more real discount rate sensitive than the income & residual capital shares because they have longer duration. The ZDPs have zero real growth rate sensitivity because they pay a fixed nominal amount whereas the income and residual capital shares clearly benefit from an increase in the real growth rate. ZDPs have negative inflation sensitivity because the redemption payment is a fixed nominal amount. Income & residual capital shares, on the other hand, have positive inflation sensitivity because the real value of the amount deducted from the fund and paid to ZDP shareholders on wind-up is reduced by an increase in expected inflation.

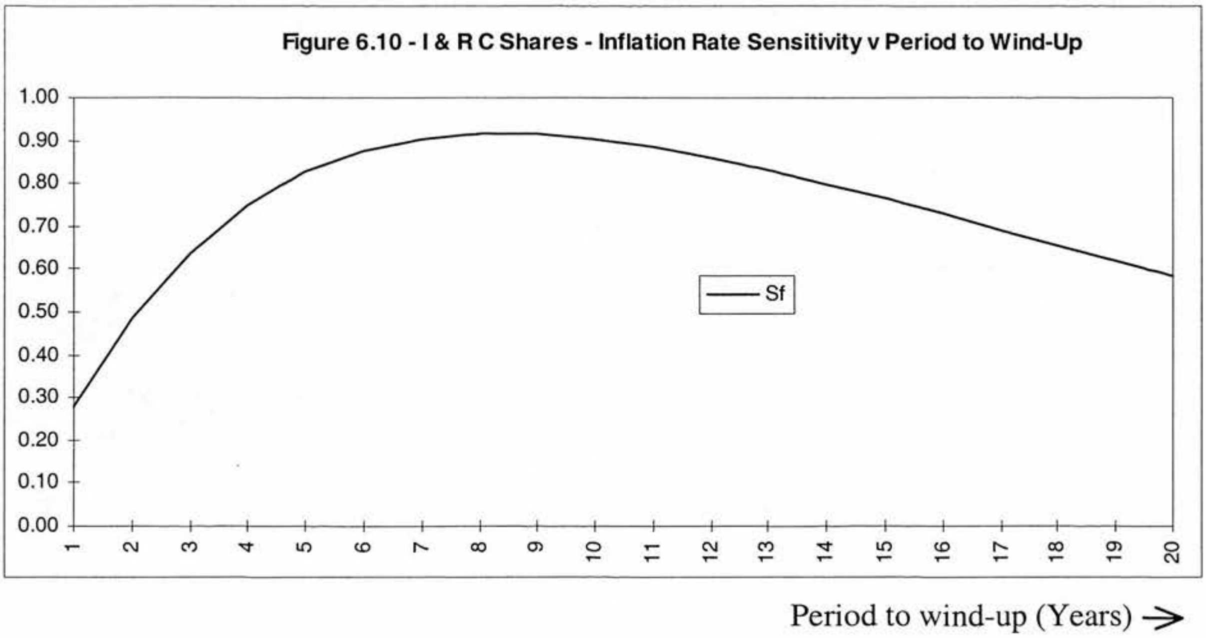
Figure 6.9 illustrates how the real discount rate sensitivity of ZDPs varies with period to wind-up.



Period to wind-up (Years) →

A graph of inflation rate sensitivity of ZDPs against period to wind-up is almost identical to Figure 6.9.

The sensitivity measures for income & residual capital shares change as the period to wind-up changes in a similar way to those for corresponding capital shares of traditional splits but for all the sensitivity measures and for all periods to wind-up, the sensitivities of income & residual capital shares are lower in magnitude. The graph of inflation rate sensitivity against period to wind-up is shown in Figure 6.10.



6.9 USES OF SENSITIVITY MEASURES

Sensitivity measures help investment analysts to understand the risks of split securities and they are already being discussed by investment banks. Cazenove & Co (1999), referring to Adams (1999), state that: "His approach enables investors to understand the sensitivity of the share price to the real discount rate, the real growth rate of the underlying assets and the rate of inflation." In particular, sensitivity measures are useful in comparing the sensitivity of different split securities to small changes in a particular underlying fundamental variable. For example, based on the approach in Adams (1999), Cazenove & Co (2000) carry out a sensitivity analysis for hypothetical 'first generation' (traditional) split structures with different levels of capital share gearing. A similar analysis is also carried out for hypothetical 'second generation' (quasi) splits.

ZDPs pay a fixed nominal amount and are normally compared with fixed-interest securities, in particular gilt strips. As a result, duration (or sensitivity to the nominal discount rate) is commonly employed in assessing risk for ZDPs (see, for example, Smith New Court, 1992).[19] The sensitivity measures in this chapter therefore develop techniques that are already used, albeit in a very limited way, in the analysis of ZDPs.

[19] The separate measurement of sensitivity to inflation and to the real discount rate, the two components of the nominal discount rate, is not considered.

The income shares and capital shares of traditional splits, and the income & residual capital shares of quasi-splits are neither fixed-interest nor equity investments. It is therefore useful to assess separately their sensitivity to inflation and to the real discount rate. This is a new approach.

Capital shares (traditional splits) and income & residual capital shares (quasi-splits) are particularly sensitive to the growth rate of the underlying fund. This is recognised by analysts, who calculate the gross redemption yield of these securities for different projected growth rates of the underlying fund. But sensitivity measures developed in this chapter enable analysts to compare not only the sensitivity of the present value of different split securities to small changes in inflation using a single statistic but also, separately, the sensitivity of the present value of different split securities to small changes in the projected real growth rate of the underlying fund in a single statistic.

Another possible use of sensitivity measures for all types of split securities is to examine the historic standard deviations of and correlations between interest rates and inflation to generate measures of 'typical' shocks, and then to show their effect on the present value of different split securities with various terms to maturity, and on the present value of split securities in high and low real interest rate/growth environments.

6.10 ACTUAL SENSITIVITY TO CHANGES OBSERVED IN THE MARKET

If the analytical approach of market operators does not mirror the approach adopted in deriving the sensitivity measures for present values, then split securities will not respond to changes in the underlying fundamental variables in the way predicted by the sensitivity measures. Random noise will also affect the actual response. In this section we test the response of split securities to changes in the underlying fundamental variables observed in the market. We must, of course, consider finite rather than infinitesimal changes in the underlying fundamental variables over finite periods of time. As the prices of split securities will drift over time (for example, towards the redemption value in the case of income shares or ZDPs) independently of changes in the underlying fundamental variables, it is not sensible simply to compare the actual percentage share price change over the period in question with the response predicted by sensitivity measures over the same period. Instead, we will consider a series of short time periods and calculate the correlation coefficient between the actual percentage change in price and the percentage change in price predicted by the sensitivity measures applied to changes in the underlying fundamental variables.

Empirical testing of sensitivity measures is fairly straightforward in the case of split securities for which real growth rate of the underlying fund is normally unimportant (income shares and ZDPs). On the other hand, empirical testing of sensitivity measures is fraught with difficulty for those split securities for which sensitivity to real growth rate of the underlying fund is important. A small (undetectable) change in long-term real dividend growth expectations for the underlying shares[20] can have a sizeable impact on the value of the underlying fund and this, in turn, can have a very considerable impact on the price of (highly-geared) capital shares or income & residual capital shares.

In 6.10.1, 6.10.2 and 6.10.3 we test the response of income shares, capital shares and ZDPs respectively to simultaneous changes in the real discount rate (*j*) and the expected inflation rate (*f*) using ten six-month periods covering the five years up to 31 December 1996. The change in the real discount rate (*dj*), measured in percentage points, is obtained from the index-linked gilt market and the change in the expected inflation inflationary expectations (*df*), also measured in percentage points, is obtained from the index-linked and conventional gilt markets.[21] The technical inaccuracies that arise in estimating future inflation from the gilt market (Deacon and Derry, 1994) tend to cancel out when determining *changes* in inflationary expectations.

The values for *dj* and *df* shown in Table 6.3 are derived from the ten year par yield for index-linked gilts (rry) with a 5% inflation assumption and the ten year par yield for conventional gilts. The data was provided by HSBC.

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[20] This translates into a much higher change in the real growth of the underlying fund up to the redemption date.

[21] Although the real redemption yield on index-linked gilts may be artificially depressed at present (Debt Management Office, 1999), the period being examined ends on 31 December 1996, well before this problem started arising.

Table 6.3: Changes in real discount rate and inflation rate

	rry	*f*	*dj*	*df*
31/12/91	4.27	5.43		
30/06/92	4.47	4.62	0.20	-0.81
31/12/92	3.92	4.22	-0.55	-0.40
30/06/93	3.53	4.23	-0.39	0.01
31/12/93	2.88	3.16	-0.65	-1.07
30/06/94	4.02	4.49	1.14	1.33
30/12/94	3.95	4.84	-0.07	0.35
30/06/95	3.85	4.65	-0.10	-0.19
29/12/95	3.56	3.85	-0.29	-0.80
28/06/96	3.90	4.02	0.34	0.17
31/12/96	3.54	4.02	-0.36	0.00

6.10.1 Income shares

There are only three traditional split capital trusts which have no warrants outstanding, have equal numbers of income shares and capital shares in issue, have uncomplicated repayment terms and for which data is available from Datastream for the whole five year period of observation. These are Aberforth, Archimedes and Lloyds Smaller Companies.

If we assume reasonably stable real dividend growth rate expectations for the income shares, we would expect a positive correlation coefficient between actual share price movements and those predicted using sensitivity measures applied to observed market changes in the real discount rate and inflationary expectations. The details are given in Tables 6.4, 6.5 and 6.6.

Table 6.4: Aberforth Income Shares

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-5.51	-0.21		104	
30/06/92	-4.94	-0.19	-0.93	96	-7.69
31/12/92	-5.09	-0.27	2.79	91	-5.21
30/06/93	-4.89	-0.27	1.98	89	-2.20
31/12/93	-5.04	-0.37	3.46	93	3.93
30/06/94	-4.98	-0.36	-6.23	92	-0.54
30/12/94	-4.67	-0.34	0.22	88	-4.35
30/06/95	-4.18	-0.30	0.53	79	-10.23
29/12/95	-4.21	-0.39	1.45	77	-2.53
28/06/96	-3.80	-0.34	-1.50	73	-5.84
31/12/96	-3.78	-0.37	1.37	78	6.90
		Correl	0.17		

Table 6.5: Archimedes Income Shares

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-4.65	-0.05		280	
30/06/92	-4.48	-0.06	-0.89	255	-8.93
31/12/92	-4.41	-0.08	2.49	220	-13.73
30/06/93	-4.34	-0.09	1.72	210	-4.55
31/12/93	-4.55	-0.11	2.92	269	28.10
30/06/94	-4.58	-0.13	-5.33	248	-7.81
30/12/94	-4.21	-0.10	0.28	255	2.82
30/06/95	-3.92	-0.12	0.44	210	-17.65
29/12/95	-3.77	-0.14	1.23	186	-11.43
28/06/96	-3.50	-0.14	-1.31	177	-4.84
31/12/96	-3.22	-0.12	1.26	186.5	5.37
		Correl	0.32		

Table 6.6: Lloyds Smaller Companies Income Shares

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-3.83	-0.69		37	
30/06/92	-3.67	-0.77	-0.21	36	-2.70
31/12/92	-3.15	-0.68	2.33	31	-15.28
30/06/93	-3.22	-0.88	1.22	30	-1.64
31/12/93	-3.16	-0.94	3.04	34	13.33
30/06/94	-2.94	-0.88	-4.86	32	-5.88
30/12/94	-2.63	-0.84	-0.10	29	-10.94
30/06/95	-2.44	-0.90	0.42	27	-7.02
29/12/95	-2.27	-0.92	1.42	27	0.00
28/06/96	-1.96	-0.83	-0.93	24	-9.43
31/12/96	-1.78	-0.81	0.70	24	-2.08
		Correl	0.33		

We do in fact observe a positive correlation coefficient for all three income shares. But the correlation coefficient for Aberforth Income shares at only 0.17 is lower than for the other two income shares. A possible reason for this is that the income shares and capital shares of Aberforth Split Level Trust can be combined (one plus one) and traded as units which would have identical characteristics to ordinary shares in a 'sister' non split level trust, Aberforth Smaller Companies Trust. There is a degree of arbitrage between the units and the ordinary shares of the sister trust. Thus the price of the units is to some degree driven by the price of the ordinary shares of the sister trust; in turn, the price of the units will to some extent drive the price of the income shares rather than vice versa.

6.10.2 Capital shares

We adopt a similar approach for the capital shares of Aberforth, Archimedes and Lloyds Smaller Companies. We would expect a positive correlation coefficient but note that the important term $S_{g*}dg*$ is not included because of the difficulties explained above.[22] The results are as follows:

Table 6.7: Aberforth Capital Shares

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-11.86	0.28		118	
30/06/92	-11.27	0.26	-2.60	126	6.78
31/12/92	-10.84	0.28	6.09	129	2.38
30/06/93	-10.29	0.21	4.23	159	23.26
31/12/93	-9.77	0.19	6.47	189	18.87
30/06/94	-9.36	0.19	-10.89	175	-7.41
30/12/94	-8.89	0.19	0.72	169	-3.43
30/06/95	-8.44	0.17	0.85	193	14.20
29/12/95	-7.94	0.15	2.31	223	15.54
28/06/96	-7.48	0.13	2.67	265	18.83
31/12/96	-7.02	0.12	2.69	271	2.26
		Correl	0.48		

Table 6.8: Archimedes Capital Shares

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-10.67	0.16		315	
30/06/92	-10.18	0.20	-2.27	283	-10.16
31/12/92	-9.76	0.23	5.52	270	-4.59
30/06/93	-9.37	0.22	3.81	300	11.11
31/12/93	-8.79	0.17	5.85	390	30.00
30/06/94	-8.48	0.19	-9.80	360	7.69
30/12/94	-8.09	0.18	0.66	408	13.33
30/06/95	-7.62	0.18	0.78	405	-0.74
29/12/95	-7.17	0.19	2.07	418	3.21
28/06/96	-6.60	0.17	-2.40	395	-5.50
31/12/96	-6.04	0.15	2.37	394	-0.25
		Correl	0.59		

[22] $g*$ is the estimated real growth rate per annum of the underlying fund up to redemption.

Table 6.9: Lloyds Smaller Companies Capital Shares

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-9.58	0.05		60	
30/06/92	-9.27	0.05	-1.95	67	11.67
31/12/92	-8.60	0.05	5.08	55.5	-17.16
30/06/93	-8.21	0.04	3.36	72	29.73
31/12/93	-7.62	0.04	5.29	79	9.72
30/06/94	-7.28	0.04	-8.63	83	5.06
30/12/94	-6.81	0.04	0.52	81	-2.41
30/06/95	-6.33	0.03	0.67	90	11.11
29/12/95	-5.84	0.03	1.81	101	12.22
28/06/96	-5.36	0.02	-1.98	115	13.86
31/12/96	-4.92	0.02	1.93	123	6.96
		Correl	-0.08		

The correlation coefficients for Aberforth capital shares and Archimedes capital shares are both positive at around 0.5 or more. The correlation coefficient for Lloyds Smaller Companies is, however, marginally negative. Such a result is not surprising given that the term $S_{g*}dg*$ has not been included in the analysis.

6.10.3 Zero dividend preference shares

There are only three quasi-split trusts which have no warrants outstanding, have equal numbers of ZDPs and income & residual capital shares in issue and for which data is available from Datastream for the whole five year period of observation. These are Edinburgh Income, Fleming Income & Capital and Invesco Recovery. The results are as follows:

Table 6.10: Edinburgh Income ZDPs

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-7.93	-7.90		40.5	
30/06/92	-7.48	-7.49	4.82	46.5	14.81
31/12/92	-7.03	-7.04	7.11	51.75	11.29
30/06/93	-6.54	-6.56	2.67	53	2.42
31/12/93	-6.08	-6.14	11.26	60	13.21
30/06/94	-5.59	-5.58	-15.09	57.25	-4.58
30/12/94	-5.11	-5.09	-1.56	59	3.06
30/06/95	-4.61	-4.62	1.48	61	3.39
29/12/95	-4.23	-4.17	5.03	72.5	18.85
28/06/96	-3.72	-3.69	-2.15	72.5	0.00
31/12/96	-3.23	-3.20	1.34	74.75	3.10
		Correl	0.79		

Table 6.11: Fleming Income & Capital ZDPs

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-9.64	-9.64		29.5	
30/06/92	-9.22	-9.24	5.88	35	18.64
31/12/92	-8.86	-8.80	8.77	42.75	22.14
30/06/93	-8.33	-8.31	3.37	42.25	-1.17
31/12/93	-7.85	-7.92	14.31	47.75	13.02
30/06/94	-7.33	-7.34	-19.47	43	-9.95
30/12/94	-6.90	-6.84	-2.06	46.5	8.14
30/06/95	-6.37	-6.37	1.99	46.5	0.00
29/12/95	-5.95	-5.94	6.94	54.25	16.67
28/06/96	-5.46	-5.45	-3.03	55.25	1.84
31/12/96	-5.00	-4.97	1.97	58.75	6.33
		Correl	0.78		

Table 6.12: Invesco Recovery ZDPs

	S_j	S_f	$S_j.dj+S_f.df$	Price	%change in price
31/12/91	-6.50	-6.52		99	
30/06/92	-6.07	-6.09	3.98	116	16.67
31/12/92	-5.62	-5.64	5.78	127	9.96
30/06/93	-5.14	-5.16	2.14	131	3.35
31/12/93	-4.67	-4.73	8.86	146	11.43
30/06/94	-4.19	-4.19	-11.61	143	-1.88
30/12/94	-3.72	-3.70	-1.17	149	3.83
30/06/95	-3.22	-3.23	1.07	154	3.19
29/12/95	-2.78	-2.77	3.51	171	10.89
28/06/96	-2.30	-2.28	-1.42	176	3.23
31/12/96	-1.80	-1.80	0.83	180	2.56
		Correl	0.77		

As expected, the correlation coefficient is high (close to 0.80) for all three ZDPs examined.

6.11 SUMMARY

The inadequacy of statistical measures of risk based on historic data for splits and the need for statistics that measure the sensitivity of split security values to changes in the underlying fundamental variables, suggests that 'sensitivity measures' may be useful in the risk analysis of split securities.

An alternative approach to risk analysis for certain types of split capital investment trust security is presented in this chapter. Following the approach of Adams and Booth (1995), sensitivity measures for these securities are derived. The sensitivity measures show the percentage change in the present value of expected future cash flows per one percentage point change in an underlying variable, for small changes in that variable. Thus a single figure can be calculated for real discount rate sensitivity, for real growth rate sensitivity and for inflation rate sensitivity of each security. This should be of considerable use to practitioners in the investment trust sector.

The formulae and graphs presented in Sections 6.7 and 6.8 show that, for all the securities considered, real discount rate sensitivity is negative and increases with period to wind-up, as expected. For traditional splits, capital shares are more real discount rate sensitive than income shares; for quasi-splits, ZDP shares are more real discount rate sensitive than income & residual capital shares. Real growth rate sensitivity is positive for all securities for which it is relevant, and again increases in magnitude with period to wind-up. For traditional splits, income shares have lower real growth rate sensitivity than capital shares. Inflation rate sensitivity is negative for income shares and for ZDP shares but is positive for capital shares and for income & residual capital shares.

Sensitivity measures help investment analysts to understand the risks of split securities. In particular, they are useful in comparing the sensitivity of the present value of different split securities to small changes in the underlying fundamental variables. But testing sensitivity measures empirically is fraught with difficulties. There is some evidence of positive correlation between the percentage change in share price and the percentage change predicted by the sensitivity measures for income shares and for capital shares. As expected, there is a very strong positive correlation coefficient for the ZDPs examined.

CHAPTER 7 - CONCLUSION

Many arguments to explain the closed-end fund discount puzzle have been advanced that are consistent with the efficient market model, including miscalculation of NAV, agency costs and tax timing. But even taken together they seem incapable of explaining all parts of the puzzle. In particular, they do not explain why discounts fluctuate so widely over time. Meanwhile, many researchers have claimed that decision rules based on discounts can systematically generate superior investment returns and in recent years attention has shifted towards explanations of the discount anomaly based upon irrational investor behaviour. Most of this work relates to US closed-end funds although a few papers concentrate on UK investment trusts.

The De Long *et al* (1990a) noise trader model asserts that stock markets generate irrational valuations because of the activities of noise traders (uninformed investors). Rational investors do not eliminate these irrational fluctuations in prices because they have finite time horizons so that arbitrage activity is risky and therefore limited. When noise traders are present, large positive returns tend to be followed by further large positive returns in the short-term.

Lee *et al* (1991) extend the De Long *et al* model, arguing that movements in the discounts of US closed-end funds reflect changes in the sentiment of individual investors who are the dominant owners of the funds. Institutional investors are the rational investors according to the model. Discount movements are cross-sectionally correlated so investor sentiment risk is systematic and priced in equilibrium. The theory requires discounts to be volatile because it is this volatility which is responsible for the underpricing of closed-end fund shares relative to their underlying net asset values (NAVs). This investor sentiment theory has received serious scrutiny in the last decade but no consensus view on the validity of the theory has yet emerged.

Research on US closed-end funds shows that fund shares are much more volatile than their underlying NAVs. This contradicts the efficient market model.[1] The importance of discount volatility as a component of total risk is emphasised but negative autocorrelation in discount changes implies that discount volatility is a less important component of total risk for longer return intervals.

Many studies of US closed-end funds report negative covariance between discount returns and NAV returns. This is not what investor sentiment theory predicts as the presence of noise traders implies positive covariance. A partial explanation may be that a rise (or fall) in NAV increases (or reduces) the contingent capital gains tax liability.[2] There is no corresponding tax argument to complicate a similar study of UK investment trusts.

Work by Hoskins (1994) on the ability to discount arbitrage[3] shows that fund share turnover (i.e. trading velocity) and standard deviation of NAV return are significant explanatory variables in a cross-sectional analysis of discount volatility for US closed-end funds. These variables relate to the ability of discount arbitrage traders to perform their operations. However, total assets, which may be regarded as a crude indicator of share liquidity, is not a significant explanatory variable.

Very little research has been carried out on excess volatility for UK investment trusts. There has been no variance decomposition study in the UK to provide evidence on the relative importance of the three components of total risk. There has also been no work analysing UK discount volatility. This thesis fills these gaps in the literature. It provides evidence of excess volatility which is mainly due to discount volatility but is also due to a positive covariance term in the variance decomposition study. This is consistent with noise trader theory. However, a cross-sectional analysis of discount volatility does not confirm that individual investor sentiment affects discount

[1] Possible reasons for excess volatility which are consistent with the efficient market model include corporate activity and changing perceptions of future management actions, but these do not seem capable of explaining the observed discount volatility.

[2] Realised capital gains are taxed, so unrealised capital appreciation increases potential tax liabilities.

[3] Discount arbitrage traders seek to buy closed-end fund shares trading at a discount and sell short the fund's underlying portfolio.

volatility. The main conclusions and shortcomings of the research are discussed in Sections 7.1 and 7.2. Areas for future research are also signposted.

An entirely separate approach to risk analysis is carried out in this thesis for split capital investment trusts because statistical measures of risk based on historical data are of limited use. The restricted life of splits means that the risk profile of their securities change over time and investment analysts are often more interested in the sensitivity of split securities to changes in the underlying fundamental variables. An alternative approach to the risk assessment of split capital investment trust securities is proposed. The main conclusions and shortcomings of this research together with suggestions for future research are discussed in Section 7.3. No previous research on split capital investment trusts has been published in the academic literature.

7.1 CONVENTIONAL TRUSTS: COMPONENTS OF TOTAL RISK

Defining total risk as variance of share return, the importance of the three components of total risk are investigated: variance of NAV return, variance of discount return and twice the covariance between NAV return and discount return. Monthly data from Datastream for the 15 year period from January 1982 to December 1996 are used for the analysis. The sample consists of the 50 largest trusts at the start of the period which survived until the end of the period.

For the average trust using monthly returns, variance of NAV return represents only about 63% of total risk, so there is clear evidence of 'excess volatility'. This contradicts the efficient market model. The corresponding figure for the Pontiff (1997) study of US closed-end funds is 74%, but his sample includes bond funds as well as equity funds and looks at the earlier period from July 1965 to December 1985.

With longer time intervals, excess volatility is reduced. For the average trust, variance of NAV return represents 68% of total risk for three-monthly intervals and 81% of total risk for six-monthly intervals.

Variance of discount return (its square root is 'discount volatility') represents about 30% of total risk for the average trust using monthly returns but this figure varies considerably across the sector. So discount volatility is normally an important component of total risk with monthly returns. Nevertheless, discount volatility for the average trust is far lower than for the average US closed-end fund in Pontiff's study. This could be explained by the earlier period of observation in the US study which includes the 1970s, a decade characterised by extreme movements in both US closed-end fund discounts and UK investment trust discounts.

With longer return intervals, the variance of discount return is a less important component of total risk, emphasising the importance of an investor's time horizon. For three-monthly returns, variance of discount return contributes only 17% of total risk for the average trust. For six-monthly returns, the contribution of the variance of discount return reduces even further to only 12% of total risk. This implies that, in assessing risk, short-term investors should be concerned with discount volatility as well as variation in NAV return whereas most investors should be primarily concerned with variation in NAV return rather than the often quoted volatility (of share return) based on monthly data. The reduction in the importance of discount volatility with longer return intervals is consistent with the De Long *et al* noise trader model because some of the transient noise will be removed with longer return intervals.

Cointegration analysis is useful in investigating the interaction between investment trust share price and corresponding NAV that leads to discount volatility. For each of the first ten trusts in the sample, it is shown that the trust's share price and the underlying NAV are indeed cointegrated. This is consistent with the negative autocorrelation in discounts reported in previous studies and explains why variance of discount return is a less important component of total risk for longer return intervals.

The covariance term is small but significantly greater than zero for the average trust, which means that discounts tend to widen (or narrow) when the underlying NAV falls (or rises). This is in stark contrast to the negative covariance term reported in

Pontiff's study of US closed-end funds. So there is a tendency for investment trust share prices to overreact to the fundamentals. This is true for monthly, three-monthly and six-monthly returns taking the whole period of observation.

Splitting the 15 year period of observation (1982-96) into three five-year sub-periods and comparing the variance components for the three periods highlights a number of issues. Firstly, there appears to be a downward trend in discount volatility over time. This probably reflects a growing awareness among market participants of successful discount trading strategies together with a reduction in transaction costs. Secondly, high (or low) discount volatility for a trust in one period is generally followed by high (or low) discount volatility for the same trust in the subsequent period. Thirdly, the covariance term is negative for the average trust in the first five-year period but significantly positive in the second and third five-year periods. Thus, trust shares were generally slow to react to changes in underlying NAVs before Big Bang in 1986 but generally overreacted to changes in NAVs in the 10 years following Big Bang. This is consistent with the Lee *et al* investor sentiment theory in that there was greater private shareholder involvement in investment trusts during the 10 year period following Big Bang compared with the previous five years. However, there was also a marked narrowing of the sector average discount over this period which is contrary to what would be expected from investor sentiment theory.

Variance of share price return, variance of NAV return and variance of discount return are all reduced if the exceptional month of October 1987 is excluded. The average correlation coefficient between NAV returns and discount returns also reduces substantially although it is still significantly positive for both the full 15 year period of observation and the relevant five year sub-period. This reduction in the importance of the covariance term suggests that the 'double whammy' effect may be partly related to liquidity as one of the main features of the 1987 Crash was the difficulty in being able to deal. It could be that the impact of noise traders on discounts is exaggerated at times of poor market liquidity. Yet the five trusts specialising in the Far East (including Japan) suffered a severe 'double whammy' effect in October 1987 and the ownership of these trusts was predominantly institutional. This suggests an institutional investor sentiment effect (a 'herd' instinct as it is sometimes described by practitioners) rather than an individual investor sentiment effect.

Evidence of the 'double whammy' effect is stronger for international and geographical specialist trusts than for UK trusts. Positive and significant correlation coefficients are observed for each of the three sub-sectors for both the later 5-year periods, with the exception of UK-invested trusts in the period 1/87 - 12/91, for which the correlation coefficient is positive but not significant. If the existence of noise traders is the underlying reason for the 'double whammy' effect, then why are they more in evidence for trusts investing overseas? This is an interesting area for further research.

One potential problem with the variance decomposition analysis is that short-term volatilities in share prices are partly driven by technical factors related to market imbalances (e.g. liquidity, bid-ask spreads). However, as the analysis is carried out using monthly (or longer) return intervals rather than daily or weekly, this should not influence the results to any great extent. Infrequent trading of shares (hence stale prices) will also bias variance estimates but again this problem reduces as the return interval increases, and it will also be less important for the larger more marketable trusts which make up the sample.

NAV is derived from the underlying portfolio which contains shares whose prices will have different degrees of staleness. So the NAV time series acts like a moving average of past 'true' prices and may be artificially smooth as a result. Stale prices for shares in the underlying portfolio will also tend to increase discount volatility and will tend to make the covariance term more negative (or less positive). These effects will be greater the higher the proportion of unquoted shares in the underlying portfolio but again will have less impact with longer return intervals.

Another shortcoming of the analysis, which is true of all variance decomposition studies, is that the variances are sample estimates of changing portfolios. As the variances are affected by all the observations in the time series, they will be imperfect estimates of the true variances. This holds true for both the variance of the trust share returns and the variance of NAV returns. Nevertheless, there is an implicit assumption that the estimation problem will affect them identically. Clearly, this is more of a problem with the analysis carried out over the entire 15 year period of observation rather than the 5 year sub-periods.

The research could be extended to investigate further the underlying reason(s) for the 'double whammy' effect. A cross-sectional regression analysis with the correlation coefficient (between NAV returns and discount returns) as the dependent variable may shine light on the underlying reason(s) for the effect. The effect seems to be less evident for UK trusts, so an obvious explanatory variable would be the *percentage of underlying assets held in the UK*. Other explanatory variables could be: *percentage of shares held by individuals* (as implied by the individual investor sentiment theory) and some measure of (or proxy for) liquidity such as *In(market value)*.

A future research question which is raised by the analysis is whether a decision rule based not only on the level of the discount but also based on exploiting the positive covariance term might be successful. Trusts on wide discounts would only be purchased when the NAV has risen x% and trusts on narrow discounts would only be sold if the NAV has fallen x%. Another decision rule which could be tested is one based not only the level of the discount but also on the level of discount volatility. According to the investor sentiment theory, trusts with high discount volatility should stand on wide discounts because the risk of discounts widening is cross-sectionally correlated and therefore priced in equilibrium. Thus, trusts on wide (or narrow) discounts would only be purchased (or sold) if their historic discount volatility were below (or above) a certain level. Both such decision rules could be tested over the years since the end of the period of observation for this thesis (31 December 1996).

To sum up, the results of the variance decomposition analysis contradict the efficient market model but are consistent with the noise trader model. There is strong evidence of excess volatility (trust share returns are more volatile than NAV returns) and a 'double whammy' effect (discount returns and NAV returns are positively correlated).

7.2 CONVENTIONAL TRUSTS: DISCOUNT VOLATILITY

The persistence of high or low discount volatility for individual trusts that was observed over the three 5 year sub-periods in the variance decomposition analysis suggests that discount volatility may be predictable. Furthermore, the considerable cross-sectional variation in discount volatility indicates that a cross-sectional regression analysis may be worthwhile. In particular, such an analysis may help to explain one of the main parts of the discount puzzle - why discounts fluctuate so widely over time.

To try and explain the cross-sectional variation in discount volatility, a sample consisting of 59 UK conventional investment trusts in continuous operation over the five years from 1 January 1992 to 31 December 1996 is examined. Discount volatility is calculated using monthly intervals. Trust attributes chosen as explanatory variables in the analysis influence discount volatility either through share price returns or through NAV returns. The chosen variables in the first category are: *trust share turnover*, *ln(market value)*, *ln(unadjusted share price)*, *percentage of shares held by individuals* and *percentage of underlying assets in the UK.* The chosen variables in the second category are: *standard deviation of NAV return, gearing* and *percentage of underlying assets which are unquoted.*

Four explanatory variables are highly significant - *trust share turnover, standard deviation of NAV return, ln(market value)* and *percentage of underlying assets which are unquoted.* With only these four explanatory variables in the regression equation, the adjusted R-square is as high as 0.74. The first two of these significant variables - *trust share turnover* and *standard deviation of NAV return* - are significant in Hoskin's study of US closed-end funds. However, the third and fourth significant variables - *ln(market value)* and *percentage of underlying assets which are unquoted* - are not even included as explanatory variables in the US study.

The likely reasons for the significance of the four explanatory variables in the current study are as follows. *Trust share turnover*, sometimes known as trading velocity, proxies for information flow which is the central driving force for share price movements. *Standard deviation of NAV return* proxies for both the ability and the need to hedge underlying net assets from the discount anomaly trader's perspective. *Ln(Market value)* proxies for marketability, and the more marketable the trust shares, the narrower the discount arbitrage bounds. *Percentage of underlying assets which are unquoted* is significant because valuations of unquoted assets tend to be historic which reduces the correlation between share price returns and NAV returns. There is no evidence that either individual investor sentiment or UK specific sentiment has any impact on discount volatility.

There is a correlation coefficient of 0.58 between *trust share turnover* and *standard deviation of NAV return*, so the coefficients for these variables may be unreliable due to multicollinearity. The Belsley condition index for the four variables confirms that multicollinearity is a problem. On examination of the variance proportions, the variables causing most collinearity are *trust share turnover* and *standard deviation of NAV return*, as expected.

To test the stability of the regression coefficients for the four variables, the period of observation is split into two equal sub-periods, and regressions are run for these two 30 month periods separately. All four explanatory variables are highly significant for both sub-periods as was the case for the full 5 year period but the coefficient of the variable *percentage of underlying assets which are unquoted* is shown to be unstable; this is not surprising as there is little cross-sectional variation in this variable.

It would be interesting to re-run the regressions using data covering the period starting at the end of the period of observation in this thesis (31 December 1996) to the present. Are the same four explanatory variables significant and is their relative importance stable over time? Another interesting area for further research would be to examine the time series behaviour of *discount volatility, trust share turnover* and *standard deviation of NAV return*. This may throw light on their stability and behaviour.[4]

[4] For example, rolling discount volatility could be examined. Drop out the first observation and add the next, calculate and then repeat procedure.

So why are discounts so volatile? The results suggest that the main driving forces for discount volatility are, firstly, new information hitting the market (trust share turnover was used in the analysis as a proxy) and, secondly, volatility of NAV returns. Discount arbitrage traders try to take advantage of discount anomalies but they have limited time horizons and their activities are restricted, especially in the case of less marketable investment trusts for which transaction costs are relatively high. Individual investor sentiment does not seem to be the cause of discount volatility.

Overall, the results of the analysis relating to conventional trusts in this thesis are consistent with the noise trader model but do not lend support to the Lee *et al* investor sentiment theory for closed-end funds.

7.3 SPLIT CAPITAL TRUSTS: SENSITIVITY MEASURES

The concepts of duration and volatility have long been applied to bonds and since the 1970s, the concept of duration has been applied to equities. Duration effectively measures the sensitivity of a security's value to changes in the nominal interest rate. A more recent development has been the derivation of 'sensitivity measures' for equities (Adams and Booth, 1995) which measure the sensitivity of equity values to small changes in a number of underlying fundamental variables, not just the nominal discount rate.

Sensitivity measures for split capital investment trusts are derived in this thesis. They show how the present value of expected future cash flows will vary as the real discount rate changes, the real growth of income or capital value of the underlying fund changes and the estimated rate of inflation changes. Thus, they can be used to compare the sensitivity of different split securities to small changes in a particular underlying fundamental variable.

Sensitivity with respect to a particular variable is approximately equal to the weighted average of the durations of the component cash flows which are sensitive to that variable. The weights are the present values (positive or negative) of the component cash flows. If the sensitivity measures are developed in continuous time, the relationship holds exactly.

In developing formulae for sensitivity measures, attention is focused on two straightforward types of split trusts: traditional splits, consisting of income shares and capital shares, and quasi-splits consisting of zero dividend preference shares (ZDPs) and income & residual capital shares.

The main shortcoming of sensitivity measures is that they do not take into account the option element of splits. In other words, they are strictly valid only in the case of split trusts for which it is very likely that the income shares (traditional splits) or ZDPs (quasi-splits) will be repaid at their full final redemption value. However, this condition will generally hold for splits launched at lower stock market levels than present, unless there has been a capital reconstruction.

For all the securities considered, real discount rate sensitivity is negative and increases with period to wind-up due to the increase in duration. Inflation rate sensitivity is negative for income shares and for ZDPs because the redemption payment is a fixed nominal amount. However, inflation rate sensitivity is positive for capital shares and for income & residual capital shares because the real value of the fixed nominal amount paid to the prior capital on wind-up is reduced by an increase in expected inflation.

Capital shares (traditional splits) and income & residual capital shares (quasi-splits) are particularly sensitive to the growth rate of the underlying fund due to the effect of gearing. The sensitivity measures enable analysts to compare the sensitivity of such securities to small changes in the estimated rate of inflation and also, separately, to compare their sensitivity to small changes in the projected real growth rate of the underlying fund.

The response of income shares, capital shares and ZDPs to simultaneous changes in the real discount rate and the expected inflation rate[5] is tested empirically using ten

[5] Changes in the real discount rate and the expected inflation rate are obtained from the gilt market.

six-month periods covering the five years up to 31 December 1996. This is carried out by calculating the correlation coefficient between the actual percentage change in price and the percentage change in price predicted by the sensitivity measures. There is a very strong positive correlation coefficient for the ZDPs examined. There is also some evidence of positive correlation between the percentage change in share price and the percentage change predicted by the sensitivity measures for income shares and for capital shares. So there is some support for the use of sensitivity measures in the risk assessment of splits. However, it is difficult to draw any firm conclusions in the case of capital shares because (undetectable) changes in the growth prospects of shares in the underlying portfolio can translate into very sharp changes in the real growth rate expectations of the underlying fund up to redemption, and hence can cause significant changes in the prices of capital shares.

Split trusts with more than two classes of shares, with bank borrowing or with other more complicated structures are not considered. However, sensitivity measures could be calculated for many of these structures. But again, the approach is strictly valid only if it is very likely that prior capital will be repaid at the full final redemption value.

An interesting area for future research would be to use a simulation approach which properly incorporates the option characteristics of split securities but which still allows the sensitivity with respect to the underlying fundamental variables to be measured. Another possible line of research would be to examine the historic standard deviations of and correlations between interest rates and inflation to generate measures of 'typical' shocks, and then to show their effect on the present value of different split securities. In this respect, impulse response simulation may be used to measure how an inflation and/or interest rate shock affects split securities.

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APPENDICES

APPENDIX 1 - The Market Model

It is well known that when a stock market goes up, most shares within that market tend to increase in price, and when a stock market goes down, most shares tend to decrease in price. Sharpe (1963) suggested that this common response to market changes could be written mathematically as:

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it} \tag{A.1}$$

where

R_{it} is the return on the ith share in period t.

R_m is the return on the market index in period t.

α_i is the constant return unique to share i.

β_i is a measure of the sensitivity of the return on share i to the return on the market index.

e_{it} is the random residual error in period t, assumed to be independently and normally distributed with zero mean and constant variance.

Equation (A.1) describes what is known as the Market Model. It requires that the only common factor affecting all securities is the return on the market index. All shares, to a greater or lesser extent, tend to move with the market.

Although β_i is generally defined in terms of monthly returns, it really reflects relationships among expectations about the values of fundamental economic variables over the long term.

It follows from equation (A.1) that:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma^2(e_i) \tag{A.2}$$

where

σ_i^2 is the variance of return on share i.

σ_m^2 is the variance of return on the market index.

$\sigma^2(e_i)$ is the variance of the error term.

The parameters α_i, β_i and $\sigma^2(e_i)$ for share i may be estimated by studying the historical relationship between the returns on share i and the returns on the market index. R_i is plotted against R_m for a number of periods (say every month for 5 years) and a 'best fit' line is drawn through the points using regression analysis. The gradient of the line is an estimate of β_i and the intercept with the y-axis is an estimate of α_i. The scatter of points about the regression line represents the residual variation in returns after removing the market effect.

The first term on the right hand side of equation (A.2), $\beta_i^2\alpha_m^2$, known as *systematic* or *market* risk, is related to fluctuations of the market as a whole and cannot be eliminated by diversification. Thus, this type of risk is important to institutional investors, as they hold diversified portfolios. The second term on the right hand side of equation (A.2), $\sigma^2(e_i)$, known as *non-systematic* or *specific* risk, can be eliminated by diversification; this type of risk is unique to the company or its industry, or is related to other factors such as company size or dividend yield.

APPENDIX 2A - The NWS Main Sub-sector Indices

The FT-SE A Investment Trust Index and the 5 NWS Main Sub-Sector Indices

FT-SE A Investment Trust
No. of Trusts (124) (130)* @ 31/12/94
No. of Securities (133) (138)* @ 1/1/95

International
(32) (34)* @ 31/12/94
(33) (35)* @ 1/1/95

Alliance
Anglo & Overseas
Bankers
Baring Tribune
British Assets
British Empire Securities
Brunner
Electric and General
English & Scottish*
Fleming Fledgeling
Foreign & Colonial
For & Col Smaller Cos
Greenfriar
Henderson Strata
Law Debenture Corporation
Majedie
Monks
Murray International*
RIT Capital Partners
St Andrew
Scottish American
Scottish Eastern
Scottish Investment
Scottish Mortgage
Scottish Value
Second Alliance
Securities Trust of Scotland
Selective Assets
TR Smaller Companies
USDC
Witan

Added
Govett High Income
London & St Lawrence

Removed
Edinburgh Investment

UK
(36) (38)* @ 31/12/94
(38) (39)* @ 1/1/95

Aberforth Smaller Companies
BZW Convertible
Drayton English & Intl.
Dunedin Income Growth
Dunedin Smaller Companies
Finsbury Growth
Fleming Claverhouse
Fleming Enterprise
Fleming Mercantile
For & Col PEP
Govett Strategic
Hoare Govett Sm Cos Index
Lowland
Malvern UK Index
Merchants
Mercury Keystone
Moorgate Smaller Cos. Inc.
Morgan Grenfell Equity Inc
Murray Income*
Shires
Smaller Companies
TR City of London
TR Property
Temple Bar
Throgmorton 1000 Sm Cos
Throgmorton Trust
Value and Income

Added
Amicable Smaller Enterprises
Dartmoor
Edinburgh Investment
Edinburgh Small Companies
For & Col Income Growth
Herald
NatWest Smaller Companies
Pilot
Schroder UK Growth Fund
Throgmorton Preferred Income
Undervalued Assets

Removed
Equity Consort*
Exeter Preferred Capital
Henderson Highland
Kleinwort Endowment Policy
London Smaller Companies
Moorgate
NB Smaller Companies
River & Merc Smaller Cos
TR High Income

European
(15) (15) @ 31/12/94
(11) (11) @ 1/1/95

Continental Assets
Fidelity European Values
Fleming Continental European
Fleming European Fledgeling
For & Col Eurotrust
For & Col German
Kleinwort Charter
Kleinwort European Privatisation
Mercury European Privatisation
Second Market
TR European Growth

Removed
European Smaller Companies
German Investment
German Smaller Companies
Paribas French

Geographical Specialists
(34) (36)* @ 31/12/94
(46) (48)* @ 1/1/95

Abtrust New Dawn
American *
Baillie Gifford Japan
Beta Global Emerging Markets
Drayton Far Eastern
Dunedin Worldwide
Edinburgh Dragon
First Philippine
Fleming American
Fleming Emerging Markets
Fleming Far Eastern
Fleming Japanese
Fleming Overseas
For & Col Emerging Mkts
For & Col Pacific
For & Col US Smaller Cos.
GT Japan
Gartmore Emerging Pacific
Govett Oriental
Kleinwort Overseas
Martin Currie Pacific
Mercury World Mining
Murray Smaller Markets*
Overseas
Pacific Assets
TR Far East Income
TR Pacific
Templeton Emerging Markets
Thornton Asian Emerging Markets
US Smaller Companies

Added
Abtrust Emerging Economies
Baillie Gifford Shin Nippon
Edinburgh Inca
Edinburgh New Tiger
Fidelity Japanese Values
Fleming Chinese
Fleming Indian
Govett Emerging Markets
HTR Japanese Smaller Cos
Morgan Grenfell Latin American
NM Smaller Australian Cos
Old Mutual South Africa
Schroder Japan Growth Fund
Scudder Latin America
Siam Selective Growth
Templeton Latin America

Removed
Govett American Smaller Cos
London American Growth
North American Gas
North Atlantic Smaller Companies

Venture & Dev. Capital
(7) (7)* @ 31/12/94
(5) (5) @ 1/1/95

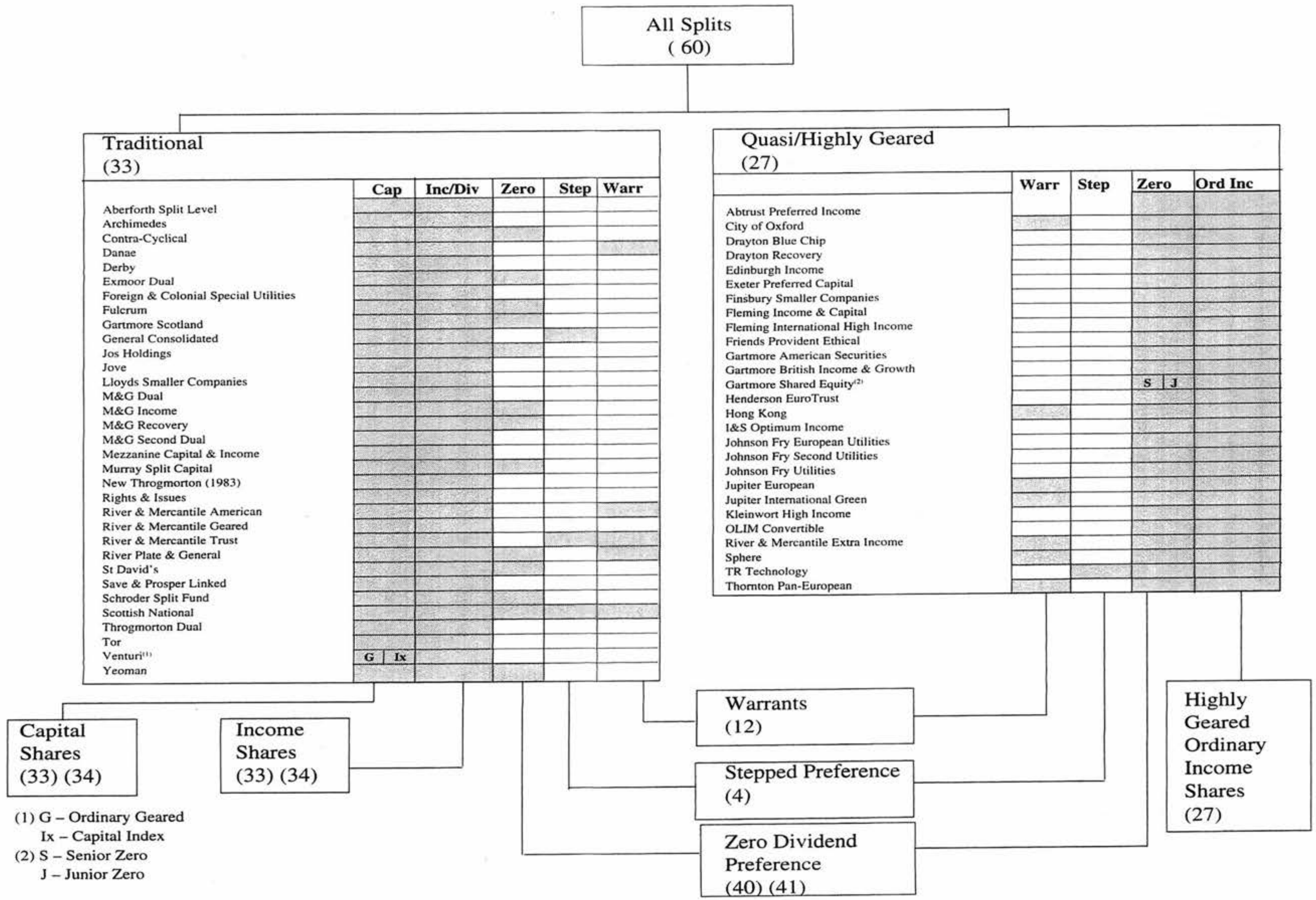
3i
Candover
Electra
For & Col Enterprise
Murray Ventures

Removed
Ivory & Sime Enterprise Capital
Second Consolidated

* B Share or Deferred Share in issue
\- Edinburgh Investment changed from the International to the UK category from 1 January 1995.

APPENDIX 2B - The NWS Split Capital Index

The NWS Split Capital Index and the 8 NWS Sub-Sector Split Indices



APPENDIX 3 - Sample for Components of Total Risk

Trust	Mnemonic
Alliance Trust PLC	atst
American Trust PLC	amts
Anglo & Overseas Trust PLC	aot
Bankers Investment Trust PLC	bnkr
Baring Tribune Investment Trust PLC	bti
British Assets Trust PLC	bset
British Investment trust PLC	bits
Brunner Investment Trust PLC	but
Dunedin Income Growth Inv Tst PLC	dig
Dunedin Smaller Co's Inv Tst PLC	dndl
Dunedin Worldwide Inv Trust PLC	dww
Edinburgh Investment Trust PLC	edin
Electric&General Investment Co PLC	elgn
English & Scottish Investors PLC	ensc
Fleming American Inv Trust PLC	fam
Fleming Claverhouse Inv Trust PLC	fcv
Fleming Continental Euro Inv Tst	fut
Fleming Far Eastern Inv Trust PLC	ffe
Fleming Japanese Inv Trust PLC	flmj
Fleming Mercantile Inv Trust PLC	fmn
Fleming Overseas Inv Trust PLC	fov
Foreign & Col Invest Trust PLC	frcl
Foreign & Col. Pacific Inv Tst PLC	fcp
Foreign & Colonial Smaller Co's PLC	fcs
G.T.Japan Investment Trust PLC	gtja
Govett Oriental Inv Trust PLC	gor
Govett Strategic Inv Trust PLC	gvs
Kleinwort Charter Inv Trust PLC	klc
Kleinwort Overseas Inv Trust PLC	kos
Merchants Trust PLC	mrch
Mercury Keystone Investment Tst PLC	mki
Monks Investment Trust PLC	mnks
Murray Income Trust PLC	mut
Murray International Trust PLC	myi
Murray Smaller Markets Trust PLC	msm
Murray Ventures PLC	mvn
Overseas Investment Trust PLC	oit
Scottish American Investment Co PLC	scam
Scottish Eastern Inv Trust PLC	scea
Scottish Investment Trust PLC	scin
Scottish Mortgage & Trust PLC	smt
Second Alliance Trust PLC	sat
Securities Trust of Scotland PLC	sts
St Andrew Trust PLC	srw
Temple Bar Investment Trust PLC	tmpl
Throgmorton Trust PLC	thrg
TR City of London Trust PLC	trcd
TR Property Investment Trust PLC	try
TR Smaller Companies Inv Trust PLC	tru
Witan Investment Co PLC	wtan

APPENDIX 4A - Test for Unit Root for P_t and A_t

ADF (order 5)

Monthly

Critical values: 5% = -2.879; 1% = -3.471

Trust	P_t	A_t	$\ln P_t$	$\ln A_t$
ATST	0.7611	0.6701	-1.2963	-1.3663
AMTS	0.2149	0.3202	-0.8728	-0.8028
AOT	-0.0602	0.0337	-1.5141	-1.5179
BNKR	0.2209	0.6353	-1.6213	-1.5347
BTI	-0.1230	0.0566	-1.9731	-1.9425
BSET	-1.5228	-1.7727	-2.6004	-2.7555
BITS	0.0643	0.1091	-1.6206	-1.5350
BUT	0.1277	0.6909	-1.4193	-1.5608
DIG	-0.9070	-0.7756	-2.7417	-2.5933
DNDL	-1.5445	-1.5758	-2.0275	-1.8953

Three-monthly

Critical values: 5% = -2.915; 1% = -3.552

Trust	P_t	A_t	$\ln P_t$	$\ln A_t$
ATST	0.6725	0.6492	-1.7214	-1.6461
AMTS	0.7152	0.8421	-0.6986	-0.5654
AOT	0.1191	0.2079	-1.6202	-1.5629
BNKR	0.7726	0.9521	-1.5395	-1.5676
BTI	0.0292	0.2606	-2.1927	-1.9442
BSET	-1.5745	-1.6954	-2.6499	-2.7589
BITS	0.0643	0.1091	-1.6206	-1.5350
BUT	0.4895	0.3931	-1.1554	-1.3477
DIG	-0.7784	-0.7189	-2.3781	-2.1777
DNDL	-1.3821	-1.4816	-1.9357	-1.7561

Six-monthly

Critical values: 5% = -2.971; 1% = -3.685

Trust	P_t	A_t	$\ln P_t$	$\ln A_t$
ATST	1.0708	1.2671	-1.6485	-1.3436
AMTS	0.3590	0.4591	-1.1096	-0.9253
AOT	-0.1034	0.0175	-1.8035	-1.6980
BNKR	0.4770	0.7151	-1.6836	-1.7442
BTI	-0.2258	0.0342	-2.3950	-2.2213
BSET	-1.6324	-1.8787	-2.3931	-2.7792
BITS	0.1103	0.1580	-1.7833	-1.6810
BUT	0.1063	0.1480	-1.3229	-1.5218
DIG	-0.9085	-0.8043	-2.6533	-2.4013
DNDL	-1.4299	-1.4716	-2.2430	-1.9555

For monthly, three-monthly and six-monthly returns we fail to reject the null hypothesis of unit root in each case. Therefore, the series are I(1) which is a necessary condition for cointegration.

APPENDIX 4B - Test for Unit Root for Residuals

$$P_t = \alpha + \beta A_t + \varepsilon_t$$

$$In(P_t) = \alpha + \beta In(A_t) + e_t$$

ADF (order 5)

Monthly

Critical values: 5% = -2.879; 1% = -3.471

Trust	ε_t	e_t
ATST	-8.4240	-8.5185
AMTS	-15.113	-13.922
AOT	-14.960	-14.614
BNKR	-15.115	-15.371
BTI	-15.499	-13.597
BSET	-16.401	-16.752
BITS	-14.346	-13.156
BUT	-15.648	-13.753
DIG	-17.002	-15.867
DNDL	-13.493	-13.329

Three-monthly

Critical values: 5% = -2.916; 1% = -3.555

Trust	ε_t	e_t
ATST	-3.6133	-4.4326
AMTS	-7.9214	-7.7688
AOT	-7.3840	-7.0358
BNKR	-8.3504	-7.6941
BTI	-7.7267	-7.4898
BSET	-7.1407	-7.4474
BITS	-6.7757	-8.0583
BUT	-8.0300	-9.3077
DIG	-7.5650	-8.4632
DNDL	-9.4097	-8.9574

Six-monthly

Critical values: 5% = -2.975; 1% = -3.696

Trust	ε_t	e_t
ATST	-4.5148	-5.3684
AMTS	-5.5654	-5.9120
AOT	-5.2826	-5.5266
BNKR	-4.2383	-4.3222
BTI	-4.3559	-4.1595
BSET	-5.8713	-6.5744
BITS	-4.5579	-5.6135
BUT	-5.1878	-5.0364
DIG	-5.7654	-5.5495
DNDL	-4.9978	-5.3702

The null hypothesis of a unit root is rejected for all the series considered. This implies that the error term from the estimated cointegrating vector is stationary. Therefore, P_t and A_t are cointegrated.

APPENDIX 5 - Sample for Cross-sectional Analysis of Discount Volatility

Trust	Mnemonic
Abtrust New Dawn Inv Trust PLC	abd
American Trust PLC	amts
Anglo & Overseas Trust PLC	aot
Bankers Investment Trust PLC	bnkr
Baring Tribune Investment Trust PLC	bti
British Assets Trust PLC	bset
British Empire Sec & General TstPLC	btem
Dartmoor Investment Trust PLC	dit
Edinburgh Dragon Trust PLC	efm
Edinburgh Investment Trust PLC	edin
Electra Investment Trust PLC	elta
English & Scottish Investors PLC	ensc
Fidelity European Values PLC	fev
Fleming American Inv Trust PLC	fam
Fleming Continental Euro Inv Tst	fut
Fleming Emerging Mkts Inv Tst PLC	fem
Fleming European Fledgling Inv Tst	fef
Fleming Far Eastern Inv Trust PLC	ffe
Fleming Japanese Inv Trust PLC	flmj
Fleming Mercantile Inv Trust PLC	fmn
Fleming Overseas Inv Trust PLC	fov
Foreign &Col EmergingMktsInvTstPLC	fct
Foreign & Col Enterprise Tst PLC	fcet
Foreign & Col Invest Trust PLC	frcl
Foreign & Col. German Inv Tst PLC	fcg
Foreign & Col. Pacific Inv Tst PLC	fcp
Foreign & Colonial Eurotrust PLC	fcu
Foreign & Colonial Smaller Cos PLC	fcs
G.T.Japan Investment Trust PLC	gtja
Gartmore Emerging Pacific Inv Tst	gtm
Govett Oriental Inv Trust PLC	gor
Govett Strategic Inv Trust PLC	gvs
INVESCO English & Intl.Trust PLC	iei
Kleinwort Charter Inv Trust PLC	klc
Kleinwort Overseas Inv Trust PLC	kos
Merchants Trust PLC	mrch
Monks Investment Trusts PLC	mnks
Moorgate Smaller Co's Inc Trust PLC	mgs
Morgan Grenfell Equity Inc Tst PLC	mge
Murray Income Trust PLC	mut
Murray International Trust PLC	myi
Murray Smaller Markets Trust PLC	msm
Overseas Investment Trust PLC	oit
RIT Capital Partners PLC	rcp
Scottish American Investment Co PLC	scam
Scottish Eastern Inv Trust PLC	scea
Scottish Investment Trust PLC	scin
Scottish Mortgage & Trust PLC	smt
Securities Trust of Scotland PLC	sts
Smaller Companies Inv Trust PLC	smc
Temple Bar Investment Trust PLC	tmpl
Templeton Emerging Markets IT PLC	tem
Throgmorton Trust PLC	thrg
TR City of London Trust PLC	trcd
TR Pacific Investment Trust PLC	trv
TR Property Investment Trust PLC	try
TR Smaller Companies Inv Trust PLC	tru
Value & Income Trust PLC	vin
Witan Investment Co PLC	wtan

APPENDIX 6A - Hurdle Rates for Traditional Split Capital Trusts

TRADITIONAL SPLIT CAPITAL TRUSTS (with no warrants outstanding)			
	Wind-up date		Hurdle
	Earliest	Latest**	Rate %
Aberforth Split Level	1.7.00	30.6.04	-36.7
Archimedes	30.9.98	30.9.03	-37.3
Derby*	1.1.99	31.12.03	-33.0
F&C Special Utilities	24.8.03	na	-12.5
Jove	1.1.95	31.12.99	-20.9
Lloyds Smaller Cos	30.4.02	na	-58.9
M&G Dual	31.12.94	31.12.96	0.0
M&G Second Dual	1.12.95	01.12.97	-87.9
MCIT 'S'	1.12.07	na	-4.0
Save & Prosper Linked	1.1.95	01.1.97	-99.9
TOR	1.8.01	any	-31.7

Source: NatWest Securities Daily NAV Service, 22 August 1996

* With Derby Trust, income shares are repayable after loan and debenture securities.

** The latest wind-up date is always used in the calculations in Chapter 6. The income shareholders will require their high level of income for the maximum length of time. The capital shareholders normally do not want their trust to be wound up early because they would incur capital gains tax. And the managers want their management fees for as long as possible.

Hurdle rate is the required annual growth rate of total assets to pay the full redemption amount of the income shares. In most cases, there is a large negative hurdle rate implying that it is extremely likely that the income shares will be repaid at their full final redemption amount.

Sensitivity measures can also be calculated for the following split capital trusts: MCIT; Rights & Issues; Venturi. They have no warrants outstanding but the repayment terms are more complicated than those for traditional split capital trusts.

APPENDIX 6B - Hurdle Rates for Quasi-split Trusts

QUASI-SPLIT TRUSTS (with no warrants outstanding)		
	Wind-up date	**Hurdle Rate %**
Abtrust Preferred	31.5.98	-16.1
Edinburgh Income	1.5.00	-3.1
Finsbury Smaller Cos	17.12.99	-36.8
Fleming Inc & Cap	3.3.02	-9.4
Fleming Int. High Inc.	5.10.96	-96.5
Friends Provident Ethical	31.12.01	-0.3
Gartmore Brit I & G	18.12.02	-0.6
Henderson Eurotrust	31.10.02	-15.2
Hong Kong	31.12.96	-84.5
HTR Income & Growth	8.3.03	-6.4
Invesco Blue Chip	10.5.98	-7.2
Invesco Recovery	18.11.98	-11.7
Ivory & Sime Optimum	26.3.97	-57.1
Johnson Fry Euro.	31.7.04	-4.6
Johnson Fry 2nd Util.	31.12.03	-4.3
Johnson Fry Utilities	31.7.03	-8.4
Kleinwort High Inc.	30.6.98	-18.9

Source: NatWest Securities Daily NAV Service, 22 August 1996

Hurdle rate is the required annual growth rate of total assets to pay the full redemption amount of the ZDPs. In most cases there is a large negative hurdle rate implying that the ZDPs will be repaid at their full final redemption amount.

Sensitivity measures can also be calculated for the following quasi-split trusts: Exeter Preferred; Gartmore Shared; Jupiter Geared. They have no warrants outstanding but the repayment terms are more complicated than those for the normal quasi-splits in the above table.

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Sensitivity measures for split-capital investment trusts

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Statistical measures of risk based on historical data are useful tools in assessing risk for conventional investment trust securities; but they are of limited use for securities of split-capital investment trusts, and an alternative approach is proposed in this paper. By differentiating formulae for the valuation of discounted cash flow, with respect to the underlying fundamental variables, 'sensitivity measures' can be derived for most securities of split-capital investment trusts. These sensitivity measures show how the present value of expected future cash flows will vary as the real discount force changes, the real force of increase in the income (or capital value) of the underlying fund changes, and the estimated force of inflation changes.

1. Introduction

Investment trusts are publicly quoted UK companies whose assets consist of a portfolio of shares or other securities. They enable investors to purchase an interest in a professionally managed fund. Ultimate responsibility for running the affairs of an investment trust lies with the board of directors, but day-to-day management is normally delegated to professional investment managers.

Split-capital investment trusts (splits) are a special type of investment trust, having more than one main class of share capital. Expected cash flows for a class of capital may be defined in nominal terms, or may depend on the income growth or capital growth of the underlying fund. Their innovative capital structures attempt to match the risk, income, and tax preferences of different types of potential investor. The great majority of splits invest entirely in the UK. They are designed to be wound up by some future date, with most splits having an original term of seven to ten years.[1] On wind-up, the trusts assets are sold, and the proceeds are used to pay off the various classes of share capital after meeting the entitlements of holders of debt, if any. However, shareholders may be given the opportunity to roll over their investment at the end of its life.

Many split-capital trusts were created in the late 1980s or early 1990s—a period characterized by high levels of inflation by recent standards. The subsequent reduction in inflation and interest rates had a considerable impact on certain types of split securities due to their inbuilt gearing. This brought home the risks inherent in many split securities, and highlighted the need for statistics that would measure the impact of changes in the underlying fundamental variables on different split securities.

[1] Most splits have a fixed wind-up date, but for some splits there may be a range of wind-up dates, and the life of others may be extended if an extension resolution is passed.

Statistical measures of risk, such as standard deviation of returns, which are normally based on historical data, are useful tools for conventional trusts but are of little use for splits. The limited life of splits means that the risk profile of their securities change over time, and analysts are often more concerned with sensitivity to the underlying fundamental variables. However, it is possible to gain an insight into the risk of many split securities using 'sensitivity measures' which were developed for equities in general by Adams & Booth (1995). These show the proportionate change in the present value of a security caused by a small change in each of the underlying fundamental variables. In other words, they show how various split securities should behave in response to changes in the underlying fundamental variables. In practice, of course, split security prices will also fluctuate with market sentiment, which we do not consider here.

Split-capital investment trusts are complex, but ultimately they just divide up the returns from the underlying portfolio of investments in a particular way. Sensitivity of the individual split securities with respect to a given fundamental variable (and weighted by their present values) should therefore add up to the sensitivity of the underlying portfolio of investments with respect to that variable.

There are two main types of split: 'traditional splits' and 'quasi-splits'. Simple valuation models are developed for traditional split securities in Section 2, and for quasi-split securities in Section 3. Estimation of the fundamental variables which determine the present values of split securities is discussed in Section 4. Section 5 demonstrates that all the basic split securities consist of one or more of three types of component cash flow. This helps in the development of sensitivity measures for split securities, which is covered in Sections 6, 7, and 8. Section 9 is the conclusion.

2. Traditional splits

A basic traditional split has its ordinary share capital divided into two distinct categories: income shares and capital shares. Holders of income shares are entitled to all the distributed income and a predetermined capital repayment on liquidation. Holders of capital shares receive no income, but are entitled to the remaining assets on liquidation after the income shares have been redeemed.[2] Thus, the traditional split allows investors to satisfy their divergent preferences for the income and capital-gain components of return from an investment fund. It can also lead to problems for the managers, however, given the inherent conflict between the interests of income shareholders and the interests of capital shareholders. The pursuit of high income normally leads to lower capital growth. The structure and management of dual-purpose funds (the US equivalents of traditional splits) are discussed by Litzenberger & Sosin (1977).[3]

Capital shares are effectively European call options on the underlying fund, with the predetermined capital repayment of income shares providing the exercise price of the

[2] The Statement of Recommended Practice for the Financial Statements of Investment Trust Companies (AITC 1995) recommends that: 'Investment management fees should be allocated between capital and revenue in accordance with the board's expected long-term split of returns, in the form of capital gains and income respectively, from the entire investment portfolio of the investment trust company'.

[3] US dual-purpose funds were dependent on a tax loophole which was closed in 1989. As a result, no new dual purpose funds may be established in the US. All the original dual-purpose funds have now been liquidated or converted to open-end status in accordance with rules laid down in their original charters.

options. The holders of income shares effectively own the underlying fund but are also writers of the call options (Ingersoll 1976). In this paper we only consider trusts in which the capital shares are deep in the money, as is normally the case (see Appendix 1), so that the call options have negligible time value, and an approach based on option theory is not required for either the capital shares or the income shares. Valuation formulae for European options, such as that of Black & Scholes, are derived by calculating the price at which an option would have to stand in the market to allow a risk-free 'hedge' between the option and the underlying asset.[4] As a result, many of the fundamental variables (see Section 4) which are important for risk-assessment purposes are not explicitly present in such formulae, and so the 'sensitivities' derived from the Black–Scholes formula (such as delta, theta, and kappa—see Hull 1997) are of limited use from the point of view of risk assessment.[5]

Even before the introduction of a gilt-strips market in 1997, risk-free spot rates[6] have been readily available from the gilt market, enabling cash flows from securities at different times to be valued at different appropriate discount rates (including risk premiums). This information could be used in valuing split securities to make discount rates vary with the term of the dividend or capital payment—and, of course, this would be an important consideration in valuing the capital-share option if it were less deeply in the money. But, in practice, redemption yields or present values based on fixed discount rates are used by investment analysts in assessing split capital securities, and this approach is convenvenient for calculating sensitivity measures in this paper. We are effectively assuming a flat spot-rate curve for gilts.

There may be other types of security in the capital structure of traditional splits—including zero-dividend preference shares (see subsection 3.1), stepped preferences shares[7], and warrants[8]. The purpose of issuing zero-dividend preference shares or stepped preference shares is to add gearing to the capital shares; the capital shares receive what is left (if anything) after all classes of prior capital have been paid their redemption values. We will not consider these more complicated capital structures further, but the approach to risk analysis in this paper may be applied to these more complicated structures, provided that the capital-share options (and warrants, if any) are deep in the money.

To simplify the discussion, we assume that the number of income shares is equal to the number of capital shares within the capital structure of the trust in question. Inflation, real growth of dividends and real growth of the underlying fund are all assumed to be constant. To simplify the mathematics, we will work in continuous time rather than in discrete time.

[4] Estimation of the parameters in the Black–Scholes model applied to splits is reasonably straightforward. For example, volatility can be estimated from the standard deviation of the return on the trust's fund, which itself can be linked to the variances and covariances of the returns on the individual investments which comprise the fund by use of mean–variance portfolio theory.

[5] One use of these Black–Scholes sensitivities from the point of view of the managers of a traditional split trust would arise if they were thinking of disposing of certain investments and buying others as replacements. The change in volatility could be estimated prior to any action, and the consequent effect on the value of the capital shares deduced by calculating kappa.

[6] A spot rate is a rate paid when money is borrowed now to be repaid at a single date in the future.

[7] Stepped preference shares pay dividends which rise at a predetermined rate, together with a fixed capital sum when the trust is wound up.

[8] Warrants are effectively long-term call options. They give the holder the right, but not the obligation, to buy shares (normally capital shares) at a predetermined price on one or more future dates.

2.1 *Income shares*

Income shares receive all the income generated by the trust's underlying portfolio, so they are suitable for investors who require a high income. Some income shares are entitled to a substantial fixed capital repayment when the trust is wound up, whereas others are more like annuities, with very little capital repayment.

Dividends from the equities in the underlying portfolio are paid out of profits which, in the medium term, tend to increase as the general level of prices increases, plus any real growth. The absolute level of dividend growth of the income shares thus depends on the level of future inflation. So, in valuing the dividends from the income shares, it is helpful to work in real terms rather than in nominal terms.

The present value of an income share is given by

$$V_0 = \int_0^m D_0 \mathrm{e}^{-(\delta-\gamma)t} \mathrm{d}t + R\mathrm{e}^{-(\delta+\phi)m},$$

where D_0 is the rate of payment of dividend per year per income share at time 0 (assumed payable continuously);
γ is the estimated real force of increase per annum in dividends (a force implies continuous compounding)
δ is the real force of discount per annum, assumed to apply to all cash flows;
ϕ is the estimated force of inflation per annum;
m is the period in years before the income shares are redeemed;
R is the redemption amount for each income share after m years.

Evaluating the integral gives

$$V_0 = D_0 \frac{1 - \mathrm{e}^{-(\delta-\gamma)m}}{\delta - \gamma} + R\mathrm{e}^{-(\delta+\phi)m}. \tag{1}$$

2.2 *Capital shares*

Capital shares receive the remaining assets of a traditional split-capital trust at the wind-up date, after all other classes of capital have received their entitlement. They receive no income, so their return depends entirely on the growth of underlying assets up to the wind-up date. Again, we assume that there is little doubt that there will be sufficient assets at the wind-up date to pay the entitlement of the income shares.

The present value of a capital share is:

$$V_0 = A_0 \mathrm{e}^{-(\delta-\zeta)m} - R\mathrm{e}^{-(\delta+\phi)m}, \tag{2}$$

where: A_0 is the value of the fund at time 0 per capital share;
ζ is the estimated real force of growth per annum of the fund;
R is the redemption amount of the income shares per capital share[9];
δ, ϕ and m are defined as in subsection 2.1.

[9] This is the same as the redemption amount per income share, because we have assumed that the number of capital shares is equal to the number of income shares.

3. Quasi-splits

Quasi-splits (also known as 'hybrid splits', 'new splits', or 'highly geared splits') always have zero-dividend preference shares (ZDPs, see below) in issue, but there is only one class of ordinary share capital, namely income-and-residual-capital shares (I&RCs). When such a trust is wound up, ZDPs are repaid first. The I&RCs effectively own the underlying fund less the discounted redemption amount of the ZDPs, but also hold a European put option on the underlying fund with exercise price equal to the redemption amount of the ZDPs. Holders of the ZDPs are writers of the put options, with the exercise price of the put equal to the redemption amount of the ZDPs; the fair value of the ZDPs is the discounted redemption amount less the value of the put option which they have written. Again, we only consider trusts in which the put option is of negligible value, as is often the case (see Appendix 2), so that option-valuation models are not required. We also assume a flat spot-rate curve for gilts as in Section 2. Quasi-splits with additional types of securities, such as stepped preference shares or warrants, are not considered.

To simplify the discussion, we assume that the number of ZDPs is equal to the number of I&RCs. Inflation, real growth of dividends, and real growth of the underlying fund are all assumed to be constant. Again, to simplify the mathematics, we will work in continuous time rather than in discrete time.

3.1 *Zero-dividend preference shares*

Zero-dividend preference shares pay a fixed capital sum on a fixed date (or on any earlier winding up) before any distribution can be made to ordinary shareholders. They have no entitlement to income, so that (importantly) there is no liability to income tax. Zero-coupon bonds (or low-coupon bonds) do not offer this tax advantage. The main influence on the prices of ZDPs is movement in the prices of gilts with similar duration.[10] The comparison with gilts has been simplified by the introduction of a gilt-strips market in 1997. This effectively means that market prices and redemption yields of zero-coupon gilts are available for comparative purposes, although tax differences with ZDPs as well as the appropriate risk premium for different ZDPs need careful consideration.

ZDPs are attractive to investors who need a fixed sum at a future point in time and are able to use their annual exemption allowance to avoid capital-gains tax. If sums of money are required at different points in time, an appropriate portfolio of ZDPs could be created.

The present value of a ZDP is given by

$$V_0 = R\mathrm{e}^{-(\delta+\phi)m}, \tag{3}$$

where: R is the redemption amount of the ZDP;
m is the period (in years) before the ZDPs are redeemed;
δ and ϕ are defined as above.

[10] Duration may be defined as the weighted average of the times of receipts of payments, where the weights are equal to the present value of those payments.

3.2 *Income-and-residual-capital shares*

Income-and-residual-capital shares (also known as 'highly geared ordinary income shares') offer high income plus all the remaining assets of a quasi-split trust at the wind-up date, after the ZDPs have received their capital entitlement. They might be looked upon as ordinary shares partly financed through borrowing. Valuing these shares requires an estimate of both income growth and growth of the underlying assets up to the wind-up date. The same real force of discount will be used for income and capital.

The present value of an I&RC is given by:

$$V_0 = \int_0^m D_0 e^{-(\delta-\gamma)t} dt + A_0 e^{-(\delta-\zeta)t} - R e^{-(\delta+\phi)m},$$

where D_0, A_0, R, m, γ, ζ, δ, and ϕ are all defined as above. Thus

$$V_0 = D_0 \frac{1 - e^{-(\delta-\gamma)m}}{\delta - \gamma} + A_0 e^{-(\delta-\zeta)m} - R e^{-(\delta+\phi)m}. \tag{4}$$

4. Estimating the fundamental variables

4.1 *Real force of discount per annum* (δ)

The real force of discount per annum is the real force of return required by investors, which is approximately equal to the required real rate of return. This will depend on the risk-free real rate of return (perhaps obtained from the index-linked gilt market) and the required risk premium.[11] It may be appropriate to use different risk premiums for different split securities. It may be argued further that different types of cash flow (e.g. bond-type income as against equity-type income) from a particular split security should be valued using different real discount rates. These complications will not be considered further in this paper, although refinements that deal with this point could be introduced into the models derived in later sections.

4.2 *Real force of increase per annum in dividends* (γ)

Force of increase in real dividends of the income shares (traditional split) or I&RCs (quasi-split) are directly related to the real growth of dividends from the underlying fund which will normally be invested in UK equities. Consideration of real dividend growth from the UK equity market may therefore provide a useful starting point in the estimation procedure. It must be remembered, however, that past dividend growth may have been financed partly by a general reduction in dividend cover, which in turn may have been influenced by factors such as taxation and dividend controls. Real dividend growth from the UK equity market has been 2.1% p.a. over the last fifty years.[12]

[11] The risk premium can be assessed using the investor's own judgement or can be estimated from similar securities, given their market prices.

[12] Based on the notional dividend of the FTSE–Actuaries All-Share Index from 1962, and on the BZW Equity Index before then. For the future, ACT and tax-credit changes may lead to an increase in retentions, and the current emphasis on 'buy-backs' as an alternative to dividends for many companies requires careful consideration.

TABLE 1

Cash flow	Present value
Stream of dividends for m years	$W_1 = D_0 \dfrac{1 - e^{-(\delta-\gamma)m}}{\delta - \gamma}$
Nominal amount R after m years	$W_2 = Re^{-(\delta+\phi)m} = Re^{-\rho m}$, where $\rho = \delta + \phi$
Value of fund after m years (per share)	$W_3 = A_0 e^{-(\delta-\zeta)m}$

4.3 *Real force of growth per annum of the fund (ζ)*

This estimate is clearly of crucial importance in the valuation of capital shares (traditional splits) and I&RCs (quasi-splits). Real growth of an appropriate index may provide a useful tentative starting point in the estimation procedure. However, growth in capital values will partly depend on changing levels of retentions and yields. The UK equity market has given real growth in capital values of 1.7% p.a. over the last fifty years.[13]

4.4 *Force of inflation per annum (ϕ)*

The market's estimate of the future rate of inflation may be derived from the gilt market. A first-order estimate is simply the redemption yield on conventional gilts less the real redemption yield[14] on index-linked gilts of similar duration. An adjustment is then required to reflect the different risk premiums on the different types of instrument and various technical factors (Deacon & Derry 1994). Investors can make further adjustments in line with their own views of the future course of the economy.

5. The component cash flows of the basic split securities

Examination of equations (1), (2), (3), and (4) shows that all the basic split securities considered in this paper can be thought of as consisting of one or more of the three component cash flows (positive or negative), with present values W_1, W_2, and W_3, shown in Table 1.

Thus:
present value (PV) of income share $= W_1 + W_2$;
PV of capital share $= W_3 - W_2$;
PV of ZDP $= W_2$;
PV of I&RC $= W_1 + W_3 - W_2$.

It is useful to treat split securities as consisting of these component cash flows when deriving their sensitivity measures in the following sections.

[13] Based on the FTSE–Actuaries All-Share Index from 1962, and on the BZW Equity Price Index before then. Since the latter index excludes small companies, the stated historic growth rate is a slight underestimate.

[14] The real redemption yield itself requires an inflation assumption.

6. Sensitivity measures

Sensitivity measures for equity investments were developed by Adams & Booth (1995). They are an extension to the concept of 'duration', which is commonly used for fixed-interest bonds and has also been applied to the analysis of equities (see, for example, Boquist *et al.* 1975). Duration is defined as the weighted average of the times of receipts from an investment, where the weights are equal to the present value of the payments. Duration is effectively a mathematical measure of the sensitivity of an investment's value to changes in the nominal discount rate in the perfect-certainty discounted-cash-flow (DCF) model.

Sensitivity measures show how the present value of expected future cash flows will vary as the fundamental variables that determine the present value vary. The analysis is valid for sensitivities to factors other than nominal discount rate. Sensitivity measures can be calculated with respect to all the underlying fundamental variables, whether they be real variables or nominal variables. An advantage of working in continuous time, with continuous forces for the underlying fundamental variables, is that the sensitivities of payments at a single time m correspond to the duration, sometimes with a change of sign.

We define sensitivity to variable k as:

$$S_k = \frac{\partial V_0}{\partial k} \cdot \frac{1}{V_0}.$$

Sensitivity to variable k can be thought of as the percentage change in the present value of the investment per one-percentage-point change in variable k.

Sensitivity measures are clearly of use in comparing the risk of different split securities. But the following should be borne in mind.

(i) The importance of a given variable in influencing the present value of a particular security over time depends not only on the sensitivity measure with respect to that variable but also on the volatility of that variable itself over time. Thus, although the sensitivity measure for a particular security with respect to variable k may be relatively low, if variable k is extremely volatile over time, it may still have an important influence on changes in the present value of that security over time.

(ii) The sensitivity measures are all partial, and do not allow for the fact that the various determinants may be interdependent. For example, real force of discount may tend to fall as expected inflation rises.

6.1 *Sensitivity measures and component cash flows*

Suppose that the cash flow from a security may be split into n component cash flows. If the present value of the ith component cash flow is B_i, then

$$V_0 = \sum_{i=1}^{n} B_i.$$

Then sensitivity with respect to a fundamental variable k is

$$S_k = \frac{\partial V_0}{\partial k} \cdot \frac{1}{V_0} = \sum_{i=1}^{n} \left(\frac{\partial B_i}{\partial k} \cdot \frac{1}{B_i} \cdot \frac{B_i}{V_0} \right)$$

TABLE 2

	Sensitivity to real discount force (δ)	Sensitivity to real growth force (γ or ζ)	Sensitivity to inflation force (ϕ)
Stream of dividends for m years	$\dfrac{me^{-(\delta-\gamma)m}}{1-e^{-(\delta-\gamma)m}} - \dfrac{1}{\delta-\gamma}$	$\dfrac{-me^{-(\delta-\gamma)m}}{1-e^{-(\delta-\gamma)m}} + \dfrac{1}{\delta-\gamma}$	0
Nominal amount after m years	$-m$	0	$-m$
Value of fund after m years	$-m$	m	0

or

$$S_k = \sum_{i=1}^{n} S_k^{B_i} \frac{B_i}{V_0}, \tag{5}$$

where $S_k^{B_i}$ is the sensitivity to variable k of the ith component cash flow. Thus:

> Sensitivity is equal to the weighted average of the sensitivities of the component cash flows, where the weights are equal to the present values of the component cash flows.

This is analagous to, but more general than, the result that duration of a portfolio of investments is the weighted average of the durations of the individual investments, where the weights are equal to the present values of the individual investments (e.g. McCutcheon & Scott 1991).

6.2 *Sensitivities of the component cash flows of splits*

The sensitivities of the component cash flows of split securities are given in Table 2. Henceforth, we will use the following notation for the cells of Table 2, where k is δ, γ, or ϕ.

S_k^1 = sensitivity of a stream of dividends for m years with respect to variable k.

S_k^2 = sensitivity of a nominal amount payable after m years with respect to variable k.

S_k^3 = sensitivity of the value of the fund after m years with respect to variable k.

The expressions in Table 2 are useful for the derivation of sensitivity measures for split securities, to which we now turn.

7. Sensitivity measures for traditional splits

7.1 *Income shares*

The present value is $W_1 + W_2$. Hence, using equation (5), we obtain

$$S_k = \frac{W_1 S_k^1 + W_2 S_k^2}{W_1 + W_2} \quad (k = \delta, \gamma, \phi). \tag{6}$$

Thus, from Tables 1 and 2, sensitivity to the real discount force is given by

$$S_\delta = \frac{\dfrac{mD_0 e^{-(\delta-\gamma)m}}{\delta-\gamma} - \dfrac{D_0(1-e^{-(\delta-\gamma)m})}{(\delta-\gamma)^2} - mRe^{-\rho m}}{D_0\dfrac{1-e^{-(\delta-\gamma)m}}{\delta-\gamma} + Re^{-\rho m}}.$$

Sensitivity to the real growth force of dividends is given by

$$S_\gamma = \frac{\dfrac{-mD_0 e^{-(\delta-\gamma)m}}{\delta-\gamma} + \dfrac{D_0(1-e^{-(\delta-\gamma)m})}{(\delta-\gamma)^2}}{D_0\dfrac{1-e^{-(\delta-\gamma)m}}{\delta-\gamma} + Re^{-\rho m}}.$$

Sensitivity to the force of inflation is given by

$$S_\phi = \frac{-mRe^{-\rho m}}{D_0\dfrac{1-e^{-(\delta-\gamma)m}}{\delta-\gamma} + Re^{-\rho m}}.$$

These sensitivity measures may be evaluated for a particular issue of income shares by choosing the appropriate values for the underlying variables (see Section 4).

7.2 *Capital shares*

The present value is $W_3 - W_2$. Hence, using equation (5), we obtain

$$S_k = \frac{W_3 S_k^3 - W_2 S_k^2}{W_3 - W_2} \quad (k = \delta, \gamma, \phi). \tag{7}$$

Thus, from Tables 1 and 2, sensitivity to the real discount force is given by

$$S_\delta = -m.$$

Sensitivity to the real growth force of the fund is given by

$$S_\zeta = \frac{mA_0 e^{-(\delta-\gamma)m}}{A_0 e^{-(\delta-\zeta)m} - Re^{-\rho m}}.$$

Sensitivity to the force of inflation is given by

$$S_\phi = \frac{mR}{A_0 e^{(\zeta+\phi)m} - R}.$$

Again, these sensitivity measures may be evaluated for a particular issue of capital shares by choosing appropriate values for the underlying variables.

7.3 *Example*

A particular traditional split trust has equal numbers of income shares and capital shares in its capital structure. We are given the following details of the trust:

rate of payment of dividend per annum per income share (D_0) = 10p;

redemption amount for each income share (R) = 50p;

value of fund at time 0 per capital share (A_0) = 200p;

period before income shares are repaid (m) = 10 years.

Estimates of the underlying fundamental variables are as follows:

real discount force (δ) = 7%;

real dividend growth force (γ) = 2%;

real growth force of the fund (ζ) = 2%;

force of inflation (ϕ) = 2.5%.

We then obtain the following sensitivity measures for the income shares and the capital shares.

	Sensitivity to real discount force	Sensitivity to real growth force	Sensitivity to inflation force
Income shares	−5.65	3.68	−1.97
Capital shares	−10	11.90	1.90

What does a figure of −5.65 for real discount force sensitivity of the income shares mean? It simply means that there is a fall of close to 5.65% in the present value of the income shares per one-percentage-point rise in the real discount force(the ratio is more exact for smaller changes). Other figures in the above table are to be interpreted in a similar way.

The income shares are less sensitive to the real discount force than are the capital shares, because they have shorter duration. The income shares are less sensitive to the real growth force than are the capital shares, because part of the return from the income shares (the redemption amount) is fixed in money terms. Income shares have negative sensitivity to the inflation force, because the redemption payment is a fixed nominal amount. Capital shares, on the other hand, have positive sensivity to the inflation force, because the real value of the amount deducted from the fund and paid to income shareholders on wind-up is reduced by an increase in inflation.

It is of interest to see how the sensitivity measures vary for different values of the period to wind-up (m), rather than fixing $m = 10$. Figures 1, 2, and 3 show how the sensitivity measures for the income shares change as the period to wind-up changes. As expected, Figs 1 and 2 show that the income shares are more sensitive to the real discount force and the real growth force (of dividends) as the period to wind-up is increased. Examination of the y-axis scale of Fig. 3 shows that the income shares are not very sensitive to inflation expectations, for all the values of m. However, sensitivity to the inflation force becomes more important if the dividend (D_0) is lower in relation to the redemption amount (R). For example, if the redemption amount is held constant—at 50p, say—then we obtain

$$S_\phi = \frac{-193.37}{7.87D_0 + 19.34}.$$

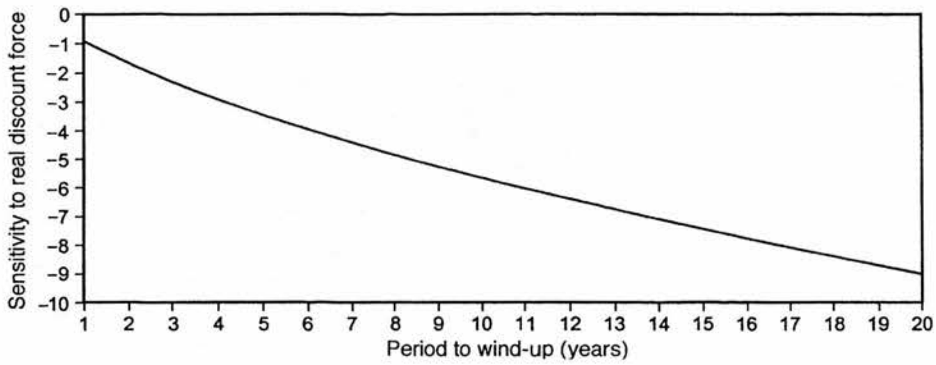

FIG. 1. Income shares: sensitivity to real discount force versus period to wind-up

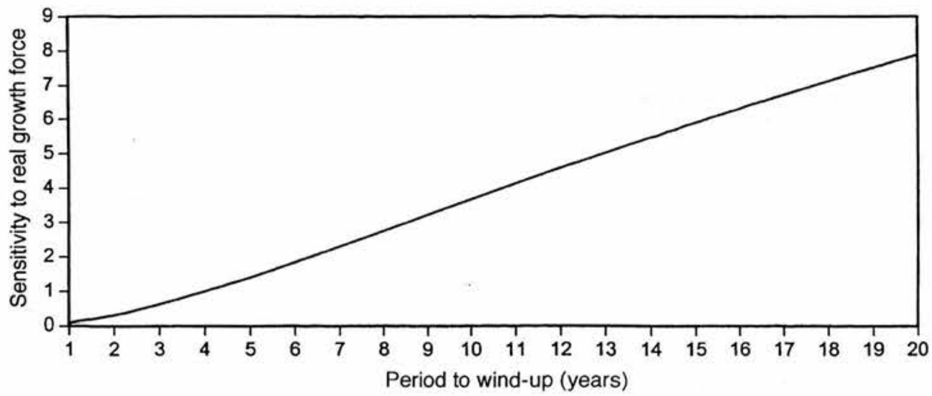

FIG. 2. Income shares: sensitivity to real growth force versus period to wind-up

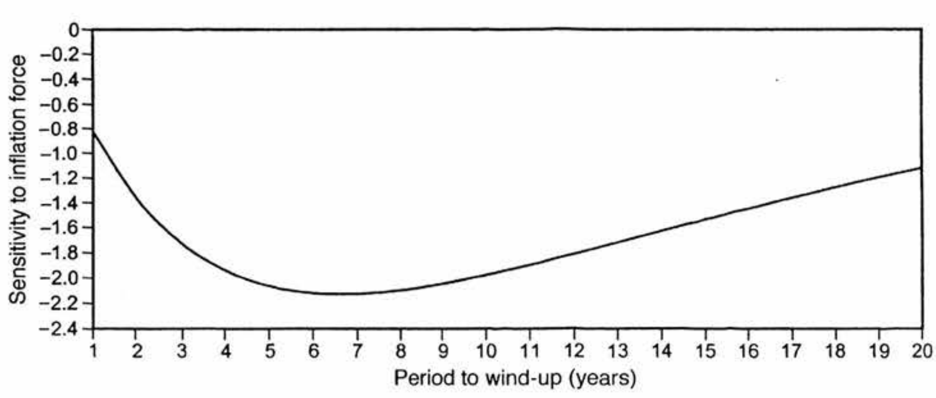

FIG. 3. Income shares: sensitivity to inflation force versus period to wind-up

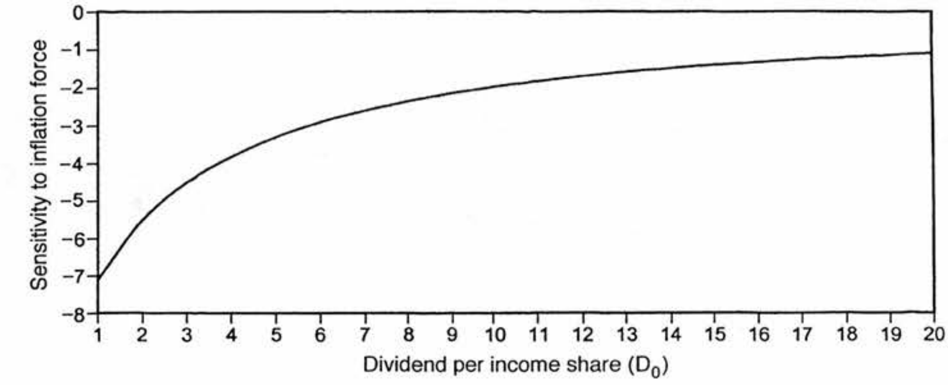

FIG. 4. Income shares: sensitivity to inflation force versus dividend

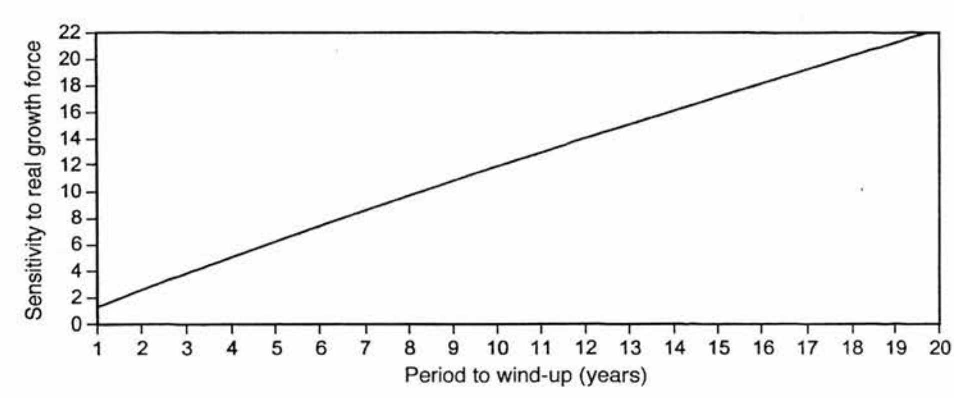

FIG. 5. Capital shares: sensitivity to real growth force versus period to wind-up

This is illustrated in Fig. 4.

Figures 5 and 6 show how the sensitivities to the real dividend growth force and the inflation force for the capital shares change as the period to wind-up changes.

The capital shares are more sensitive to the real discount force and the real growth force (of the fund) as the period to wind-up is increased. Examination of the y-axis scale of Fig. 6 shows that the capital shares are not very sensitive to inflation expectations, for all values of m. However, sensitivity to the inflation force becomes more important if the redemption amount (R) of the income shares is higher in relation to the value of the fund per share (A_0). For example, if the value of the fund per share at time 0 is held constant—say at 200p—then we obtain

$$S_\phi = \frac{10R}{313.66 - R}.$$

This is illustrated in Fig. 7.

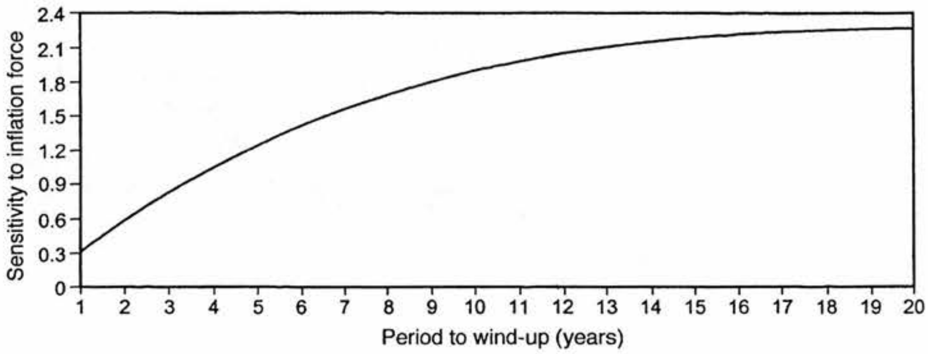

FIG. 6. Capital shares: sensitivity to inflation force versus period to wind-up

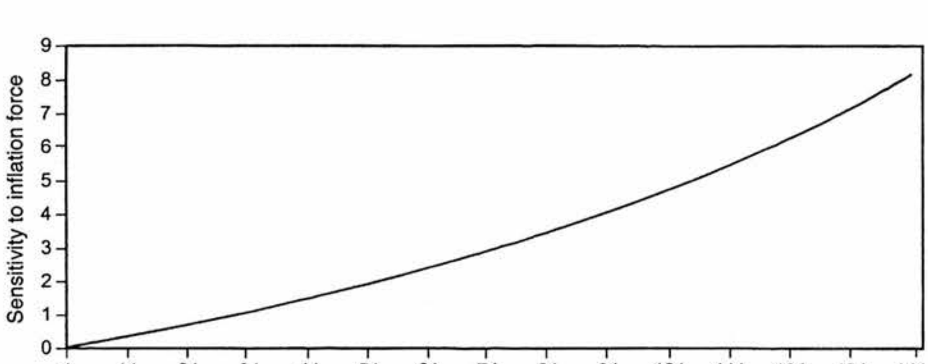

FIG. 7. Capital shares: sensitivity to inflation force versus redemption amount per income share

8. Sensitivity measures for quasi-splits

8.1 *Zero-dividend preference shares*

The present value is W_2. Hence

$$S_k = S_k^2 \quad (k = \delta, \phi). \tag{8}$$

Thus, using Table 2, sensitivity to the real discount force is given by

$$S_\delta = -m,$$

and sensitivity to the estimated force of inflation is given by

$$S_\phi = -m.$$

8.2 *Income-and-residual-capital shares*

The present value is $W_1 + W_3 - W_2$. Hence equation (5) gives

$$S_k = \frac{W_1 S_k^1 + W_3 S_k^3 - W_2 S_k^2}{W_1 + W_3 - W_2} \quad (k = \delta, \gamma, \zeta, \phi). \tag{9}$$

Sensitivity measures may then be derived, using Tables 1 and 2, and evaluated for a particular issue of I&RCs by choosing appropriate values for the underlying fundamental variables.

A particular problem arises in the case of I&RCs. The estimated real growth force of dividends (γ) and the estimated real growth rate of the underlying fund (ζ) may not be independent. It is within the power of the trust managers to increase γ, but in doing so they may reduce ζ. To simplify the discussion in the following example, we will consider only sensitivity to a single growth force by setting $\zeta = \gamma$. Broadly speaking, the assumption here is that the dividend yield of the underlying fund remains constant.

8.3 *Example*

A particular quasi-split trust has equal numbers of ZDPs and I&RCs in its capital structure. We are given the following details of the trust:

rate of payment of dividend per year per I&RC at time 0 (D_0) = 10p;

redemption amount of a ZDP (R) = 50p;

value of fund at time 0 per I&RC (A_0) = 200p;

period before the ZDPs are redeemed (m) = 10 years.

Estimates of the underlying fundamental variables are as follows:

real discount force (δ) = 7%;

real dividend growth force (γ) = 2%;

real growth force of the fund (ζ) = 2%;

force of inflation (ϕ) = 2.5%.

We then obtain the following sensitivity measures using equations (8) and (9).

	Sensitivity to real discount force	Sensitivity to real growth force	Sensitivity to inflation force
ZDPs	-10	0	-10
I&RCs	-7.64	8.71	1.07

A figure of -10 for the real discount force sensitivity of the ZDPs means that there is a fall of ~~10force~~. Other figures in the above table are to be interpreted in a similar way.

The ZDPs are more sensitive to the real discount force than arethe I&RCs because they have longer duration. The ZDPs have zero sensitivity to the real growth force, because they pay a fixed nominal amount, whereas the I&RCs clearly benefit from an increase in the real growth force. ZDPs have negative inflation sensitivity, because the redemption payment

close to 10% in the present value of the ZDPs per one-percentage-point rise in the real discount force (the ratio is more exact for smaller changes).

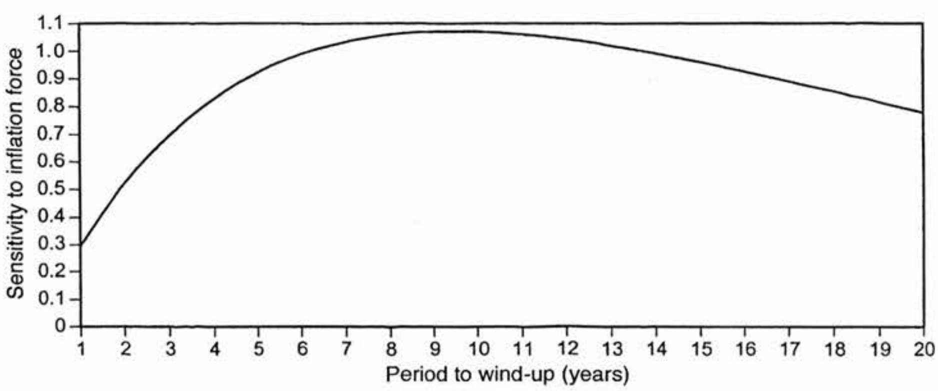

FIG. 8. Income an residual capital shares: sensitivity to inflation force versus period to wind-up

is a fixed nominal amount. I&RCs, on the other hand, have positive inflation sensitivity, because the real value of the amount deducted from the fund and paid to ZDP shareholders on wind-up is reduced by an increase in expected inflation.

The sensitivity measures for I&RCs change as the period to wind-up changes, in a way similar to that for corresponding capital shares of traditional splits; but, for all sensitivity measures and for all periods to wind-up, the sensitivities of I&RCs are lower in magnitude. The graph of sensitivity to inflation force against period to wind-up is shown in Fig. 8.

9. Conclusion

Statistical risk measures based on historical data, such as total risk, beta, and specific risk, are of limited use for split-capital investment trusts. The restricted life of these trusts means that the risk profile of their securities changes over time, and investment analysts are often more interested in the sensitivity of split securities to changes in the underlying fundamental variables.

An alternative approach to risk analysis for certain types of security in split-capital investment trusts is presented in this paper. Following the approach of Adams & Booth (1995), sensitivity measures for these securities are derived. The sensitivity measures show the percentage change in the present value of expected future cash flows per one-percentage-point change in an underlying variable. Thus, for each security, a single figure can be calculated for sensitivity to the real discount force, the real growth force, and the inflation force. This should be of considerable use to practitioners in the investment trust sector.

The formulae and graphs presented in Sections 7 and 8 of this paper show that, for all the securities considered, sensitivity to the real discount force is negative and increases with period to wind-up, as expected. For traditional splits, capital shares are more sensitive to the real discount force than income shares; for quasi-splits, z¢ero-dividend preference shares are more sensitive to the real discount force than income-and-residual-capital shares. Sensitivity to the real growth force is positive for all securities for which it is relevant, and again increases in magnitude with period to wind-up. For traditional splits, income shares have lower sensitivity to the real growth force than do capital shares. Sensitivity to the inflation force is negative for income shares and for ZDPs, but is positive for capital shares and for I&RCs.

Sensitivity measures will help investment analysts to understand the risk of a given split security. An obvious use is in comparing the sensitivity of different split securities to small changes in a particular underlying fundamental variable. Another possible use is to examine the historic standard deviations of and correlations between interest rates and inflation to generate measures of 'typical' shocks, and then to show their effect on the value of different split securities with various terms to maturity, and on the value of split securities in environments of high and low growth and real interest rates.

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Appendix A.

Hurdle rate is the minimum annual growth rate in total assets required to enable repayment of income shares at their full final redemption value. In most cases, there is a large negative hurdle rate, implying that it is extremely likely that the income shares will be repaid at their full final redemption value.

Sensitivity measures can also be calculated for the following split capital trusts: MCIT; Rights & Issues; Venturi. They have no warrants outstanding, but the repayment terms are more complicated than those for traditional split-capital trusts.

Appendix B.

Hurdle rate is the minimum annual growth rate in the total assets required to enable repayment of ZDPs at their full final redemption value. In most cases there is a large negative hurdle rate, implying that the ZDPs will be repaid at their full final redemption value.

Sensitivity measures can also be calculated for the following quasi-split trusts: Exeter Preferred; Gartmore Shared; Jupiter Geared. They have no warrants outstanding but the repayment terms are more complicated than those for the normal quasi-splits in the above table.

TABLE A1

Traditional split-capital trusts (with no warrants outstanding)

	Wind-up date		Hurdle rate %
	Earliest	Latest	
Aberforth Split Level	01 07 2000	30 06 2004	−36.7
Archimedes	30 09 1998	30 09 2003	−37.3
Derby*	01 01 1999	31 12 2003	−33.0
F&C Special Utilities	24 08 2003	na	−12.5
Jove	01 01 1995	31 12 1999	−20.9
Lloyds Smaller Cos	30 04 2002	na	−58.9
M&G Dual	31 12 1994	31 12 1996	0.0
M&G Second Dual	01 12 1995	01 12 1997	−87.9
MCIT 'S'	01 12 2007	na	−4.0
Save & Prosper Linked	01 01 1995	01 01 1997	−99.9
TOR	01 08 2001	any	−31.7

* With Derby Trust, income shares are repayable after loan and debenture securities.
Source: NatWest Securities Daily NAV Service, 22 August 1996.

TABLE B1

Quasi-split trusts (with no warrants outstanding)

	Wind-up date	Hurdle rate %
Abtrust Preferred	31 05 1998	−16.1
Edinburgh Income	01 05 2000	−3.1
Finsbury Smaller Cos	17 12 1999	−36.8
Fleming Inc & Cap	03 03 2002	−9.4
Fleming Int. High Inc.	05 10 1996	−96.5
Friends Provident Ethical	31 12 2001	−0.3
Gartmore Brit I & G	18 12 2002	−0.6
Henderson Eurotrust	31 10 2002	−15.2
Hong Kong	31 12 1996	−84.5
HTR Income & Growth	08 03 2003	−6.4
Invesco Blue Chip	10 05 1998	−7.2
Invesco Recovery	18 11 1998	−11.7
Ivory & Sime Optimum	26 03 1997	−57.1
Johnson Fry Euro.	31 07 2004	−4.6
Johnson Fry 2nd Util.	31 12 2003	−4.3
Johnson Fry Utilities	31 07 2003	−8.4
Kleinwort High Inc.	30 06 1998	−18.9

Source: NatWest Securities Daily NAV Service, 22 August 1996

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Sensitivity measures for equity investments

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By differentiating a discounted cash flow valuation formula (expressed in real terms), with respect to the underlying *fundamental* real variables, we derive 'sensitivity measures' for equity investments. These show how the present value of expected future dividends will vary as the real discount rate changes and the long-term real dividend growth rate changes. In both cases, sensitivity depends on dividend yield: the lower the dividend yield, the more sensitive the stock to changes in both the real discount rate and the real growth rate. We also derive a total differential model which allows for simultaneous changes in both the real discount rate and the real growth rate, and briefly compare this model with the model of Leibowitz *et al.* for equity duration.

1. Introduction

Duration is defined as the weighted average of the times of receipts from an investment, where the weights are equal to the present values of the payments. Duration is effectively a mathematical measure of the sensitivity of an investment value to changes in the discount rate in the perfect-certainty discounted cash flow (DCF) model.

The concept of duration was developed by Macaulay (1938). While most of the early work on duration applied the concept to the analysis of fixed-interest bonds, it can also be applied to the analysis of equities. This has been done by Casabona *et al.* (1984) and Gould & Sorensen (1986) using a constant growth rate for dividends in the DCF model (the Gordon–Shapiro model).

A more recent development by Leibowitz *et al.* (1989) has been the recognition that inflation—a major component of nominal interest rates and hence the discount rate—also affects the earnings/dividend growth rate. Their equity duration model indicates that the sensitivity of equity prices to nominal interest rates will be far less than that indicated by the duration measure derived from the traditional dividend discount model. This is because movements in nominal interest rates often occur at the same time as movements in expected nominal dividend growth rates. On the other hand, movements in interest rates which are independent of movements in expected dividend growth can have a significant effect on equity prices.

The Leibowitz *et al.* model concentrates on a DCF approach with cash flows defined in nominal terms. In using a dividend discount model, however, it is important that the financial variables be chosen to facilitate ease of estimation and stability of the model. As equities are generally regarded as real investments, it is sensible to analyse such investments using real variables rather than nominal variables.

In this paper we derive 'sensitivity measures' which show the extent to which the present value of expected future dividends is affected by changes in the underlying real variables. The analysis is valid for sensitivities to factors other than the real discount rate. Sensitivity measures can be calculated with respect to all the variables which determine the present value of an investment. Using these sensitivity measures derived from a DCF model expressed in real terms, we will develop further the work of Leibowitz *et al.*

It should be emphasized that a fluctuation in present value does not necessarily correspond to a fluctuation in price. There is, for example, much evidence of 'excess volatility' of equity prices both for the US stock market (Shiller 1981; LeRoy 1990) and for the UK stock market (Bulkley & Tonks 1989; Mills 1993). Nevertheless, sensitivity measures are useful since they give an indication of the way in which changes in the fundamental real variables affect the present value of the expected income stream and hence the way in which such changes *should* affect the price. They also provide a mathematical justification for the importance of dividend yield in equity risk analysis.

Section 2 of this paper looks at the valuation of equities in real terms, and Section 3 discusses sensitivity measures for equities. Section 4 discusses the importance of dividend yield in the analysis of risk and return. Section 5 is the conclusion.

2. DCF valuation (in real terms) of equity share investments

The dividends from an equity share are paid out of profits which tend to increase as the general level of prices increases. The absolute level of dividend growth therefore depends on the level of future inflation. In the very long term, *real* dividend growth for a domestic equity market may be reasonably predictable although Wilkie (1995a,b) shows that real dividend growth for the UK equity market has been stable around different levels for significant periods.

The Gordon–Shapiro model which assumes constant nominal growth of dividends can be expressed in real terms by simply redefining the variables to be in real terms (e.g. Adams *et al.* 1993: Ch. 4). Such a model has the advantage of being easy to understand but probably does not represent the reality of the valuation process sufficiently well. Most companies will tend to smooth dividends, thus trying at least to maintain them even when earnings are reduced due to the effects of recession. Nevertheless, as an economy comes out of (or moves into) recession, investors would expect abnormally high (or low) short-term dividend growth. Thus, for a limited period, say 5 years, analysts may wish to assume a particular real rate of growth which will allow for any temporary suppression or expansion of earnings and dividends due to the prevailing economic circumstances and particular circumstances of the company in question, at the time of valuation. Beyond this short-term consideration, one would not expect dividend growth in future years to take the smooth path assumed by a constant real growth rate of dividends. But good years can be expected to even out bad years, and the particular timing of good and bad years is of marginal significance.

Having specified the framework in which analysts should estimate the future dividends to be discounted, we now propose the model. We will assume that dividends are paid annually and that the next dividend is due in exactly one year. Taxation is ignored. We use the following notation:

V_0 is the value of the equity share;
D_1 is the estimated dividend payable one year from now;
g_1 is the estimated real dividend growth rate per annum for the first m years after D_1 is paid;
g_2 is the estimated long term real dividend growth rate per annum (after $m + 1$ years from now);
j is the real discount rate.

Then

$$V_0 = \frac{D_1}{1+j}\left[\sum_{k=0}^{m-1}\left(\frac{1+g_1}{1+j}\right)^k + \left(\frac{1+g_1}{1+j}\right)^m \sum_{k=0}^{\infty}\left(\frac{1+g_2}{1+j}\right)^k\right].$$

To obtain a finite value for the equity, we require g_2 to be less than j; but it is not necessary for g_1 to be less than j. Thus

$$V_0 = D_1\left[\frac{1}{j-g_1} - \frac{g_1-g_2}{(j-g_1)(j-g_2)}\left(\frac{1+g_1}{1+j}\right)^m\right], \tag{2.1}$$

where $j > 0$ and $j > g_2 > -1$.

Long-term real dividend growth

The appropriate long-term real rate of dividend growth should be compatible with maintaining current (or recent) levels of dividend cover. Simply looking at past dividend growth over the medium term, say twenty years, may provide a poor estimate of long-term dividend growth. For example, past dividend growth may have been financed partly by reducing dividend cover, which in turn may be influenced by factors such as taxation and dividend controls, at the expense of future long-term dividend growth. Looking backwards over very long periods may give better indications of long-term sustainable real growth rates for dividends.

It is wrong simply to choose the long-term real rate of growth of the economy as a long-term real dividend growth rate, even when corporate earnings are generated entirely from the domestic economy. The economy grows because of additions to physical capital and because capital and labour become more productive or are used more efficiently, perhaps due to innovation. Dividends grow partly because capital within a company can be used more efficiently, but largely because of the retention of profits for reinvestment in the company. There will be some relationship, although not a direct one, between these factors which cause dividend growth and the factors which cause more general economic growth. These issues are discussed in Arthur (1993).

The future long-term rate of dividend growth could be estimated by considering the marginal return on capital of the company (or perhaps sector) and the average level of retentions by the company. A company with a high level of retentions should, other things being equal, have a high level of expected future profits growth and hence dividend growth.

Real discount rate

The real discount rate is the real rate of return required by investors. This will depend on the risk-free real rate of return (obtained from the market for long-term index-linked gilts) and the required risk premium.

Much work has been done, both in the UK and in the US, to determine the apparent historical risk premium required by equity investors. This work has tended to take the implied risk premium to be the difference between equity returns and returns from instruments such as treasury bills or government bonds (e.g. Grubb 1993; Dimson 1993). There are a number of difficulties with this approach (Fitzgerald 1992; Wilkie 1994). For the UK, treasury bills have not been a perfect hedge against inflation. The structure of short-term interest rates was distorted by the building-society cartel and the monetary policy pursued in the 1970s. For much of the last 30 years, real returns from conventional gilts have been affected by unanticipated inflation. Furthermore, equity returns have been influenced by the removal of dividend controls and the rerating of equities arising from the perceived reduction in their risk compared with conventional gilts in an inflationary environment.

A more appropriate way to determine the equity risk premium, and one which can be employed directly for individual shares, is to compare the real rate of return from index-linked gilts with the anticipated real return from equities. The latter could be estimated as the current dividend yield plus expected long-term real dividend growth.

Impact of inflation

Note from equation (2.1) that inflation has no effect on the value of the equity if it neither affects real dividend growth nor the real discount rate. But, in practice, dividend increases will not respond immediately to inflation. An increase in inflation is likely to lead to an increase in company profits, at some stage, but this will not feed through immediately into increased nominal dividend growth. The transmission mechanism of monetary policy may also have a beneficial short-term effect on equity prices (Pepper 1994). Indeed, the Bank of England uses equity prices as one of its leading indicators of inflation.

In the above model, the best way to allow for the lagged way in which increased (or reduced) inflation feeds through into dividends is by adjusting g_1. Thus, an increase in inflation, for example, may lead to a reduction in real dividend growth in the short term, so there will be a reduction in g_1. There may then be a 'catching up phase' before dividends return to their long-term growth path. This could be allowed for by having a second short-term dividend growth rate before dividends return to their long term growth rate. Fuller & Hsia (1984) have suggested a simplified approach to the three-stage nominal DCF model in which the second period is a transition period during which the growth rate changes linearly from g_1 to g_2. For the purpose of the discussion in hand, however, little is added by complicating the analysis in this way.

Changes in the outlook for inflation may alter the risk perception of investors towards equity shares. In the DCF model, this would be incorporated by changing the discount rate via the risk premium.

3. Sensitivity measures

The extent of the proportionate change in the present value of an equity share caused by a change in the valuation real discount rate is of particular interest to short-term investors. Managers of non-life insurance funds, for example, are concerned about sensitivity to the real discount rate, since a short term fall in equity values could render a non-life insurance company insolvent, depending on the level of fixed-interest investment and free reserves. In general, an investor who is matching long-term real liabilities with an equity investment would be less concerned with changes in the real discount rate than with changes in prospective real dividend growth rates. Any change in the discount rate should affect the value of the investor's liabilities in the same way as it affects the value of the assets. It may not be possible, however, to absorb changes in real interest rates into the liability valuation assumptions of a life fund or pension fund because of valuation restrictions. As a result, long-term investors should also be interested in the sensitivity to real interest rates.

Both short-term and long-term investors will be concerned with the sensitivity of equity values to changes in long-term real growth rates of dividends. For short-term investors, this is due to the fall (or rise) in capital values caused by a fall (or rise) in expected long-term dividend growth.

Sensitivity to the real discount rate

For an equity share investment, the *sensitivity to the real discount rate* is defined as

$$S_j = -\frac{\partial V_0}{\partial j}\frac{1}{V_0}.$$

Differentiating equation (2.1) with respect to j gives

$$\frac{\partial V_0}{\partial j} = D_1\left[\frac{-1}{(j-g_1)^2} + \frac{(g_1-g_2)(1+g_1)^m}{(j-g_1)(j-g_2)(1+j)^m}\left(\frac{1}{j-g_1}+\frac{1}{j-g_2}+\frac{m}{1+j}\right)\right].$$

Hence

$$S_j = \frac{1-\dfrac{g_1-g_2}{j-g_2}\left(\dfrac{1+g_1}{1+j}\right)^m\left(1+\dfrac{j-g_1}{j-g_2}+m\dfrac{j-g_1}{1+j}\right)}{(j-g_1)\left[1-\dfrac{g_1-g_2}{j-g_2}\left(\dfrac{1+g_1}{1+j}\right)^m\right]}. \tag{3.1}$$

Note that, if we set $g_1 = g_2$ $(=g)$ in equation (3.1), we obtain the following simple expression for sensitivity to the real discount rate under constant real growth

$$S_j = \frac{1}{j-g}, \tag{3.2}$$

where $j > 0$ and $j > g > -1$. Equation (3.2) provides a good indicator of the sensitivity of an equity to the real discount rate, with appropriate values of j and g, in stable economic conditions. Note that, the higher the estimated real dividend growth rate, the higher the sensitivity to changes in real discount rates.

Sensitivity to the real dividend growth rate

The effect on the value of an equity share caused by a change in the long-term growth rate g_2 is of more interest than the effect on the value caused by a change in the short-term growth rate g_1. There are a number of possible reasons for a change in g_1, two of the main ones being as follows.

- A quicker (or slower) than anticipated movement out of or into recession, or factors specific to the company, may cause g_1 to be higher (or lower) than previously estimated. The likely effect of this is to shorten (or lengthen) m in a way which tends to cancel out the effect of the change in g_1. The difficulty of separating the effects of changes in m and g_1 would make sensitivity measures for g_1 of spurious accuracy.
- A change in distribution policy affects g_1, but this is likely to cause an offsetting effect on g_2.

We now derive a sensitivity measure with respect to the long-term real growth rate of dividends. Define the *sensitivity to long-term real dividend growth* as

$$S_{g_2} = \frac{\partial V_0}{\partial g_2}\frac{1}{V_0}$$

Differentiating equation (2.1) with respect to g_2 gives

$$\frac{\partial V_0}{\partial g_2} = \frac{D_1}{(j-g_2)^2}\left(\frac{1+g_1}{1+j}\right)^m,$$

so that

$$S_{g_2} = \frac{1}{j-g_2} - \frac{1}{(j-g_1)\Big/\left[\left(\dfrac{1+j}{1+g_1}\right)^m - 1\right] + j - g_2}. \tag{3.3}$$

Noting that $[(1+j)/(1-g_1)]^m - 1$ and $j - g_1$ are of the same sign, and since $j > g_2$, it may be observed from equation (3.3) that S_{g_2} increases as g_2 increases. Thus, equities which have high expected long-term real growth rates for dividends are more vulnerable to falls in the expected long-term real dividend growth rate.

By setting $g_1 = g_2 = g$ in (2.1), we can easily derive the sensitivity to real dividend growth when constant real growth is assumed:

$$S_g = \frac{1}{j-g}, \tag{3.4}$$

where $j > 0$ and $j > g > -1$.

Again, equation (3.4) provides a good indicator of the sensitivity to long-term real dividend growth for an equity, with appropriate values of j and g, in stable economic conditions. Thus, sensitivity to real dividend growth increases as the real discount rate falls and the real growth rate of dividends rises.

We will restrict the discussion hereafter to the case of a constant growth rate, i.e. with $g_1 = g_2 = g$.

Simultaneous changes in the expected real dividend growth rate and the real discount rate

The effect on an equity share value of simultaneous changes in j and g can be assessed using the total differential.

$$\mathrm{d}V_0 = \frac{\partial V_0}{\partial j}\,\mathrm{d}j + \frac{\partial V_0}{\partial g}\,\mathrm{d}g$$

$$\Rightarrow \quad \frac{\mathrm{d}V_0}{V_0} = \frac{1}{V_0}\frac{\partial V_0}{\partial j}\,\mathrm{d}j + \frac{1}{V_0}\frac{\partial V_0}{\partial g}\,\mathrm{d}g = -S_j\,\mathrm{d}j + S_g\,\mathrm{d}g. \tag{3.5}$$

Therefore, from equations (3.2) and (3.4),

$$\frac{\mathrm{d}V_0}{V_0} = \frac{\mathrm{d}g - \mathrm{d}j}{j-g}.$$

Dependency between the real discount rate and the expected real dividend growth rate

It is possible that there is a relationship between the real discount rate and the expected real dividend growth rate. The real discount rate depends partly on the perceived risk which, in turn, may depend partly on the level of risk indicated by the sensitivity measures, suggesting that shares with high estimated real dividend growth rates could be associated with high discount rates.

As a simple example, suppose that we have the cross-sectional relationship

$$j = a_0 + a_1 g,$$

where $0 < a_1 < 1$ and a_0 and a_1 are constant across different shares at a given time. Then $\mathrm{d}j = a_1\,\mathrm{d}g$, and therefore

$$\frac{\mathrm{d}V_0}{V_0} = \frac{(1-a_1)\,\mathrm{d}g}{j-g}.$$

Thus, in this example, a change in expectations for future real dividend growth will not fully translate into a corresponding proportionate change in present value.

Comparison with the Leibowitz et al. model

Leibowitz *et al.* (1989) concentrate on duration which, assuming continuous cash flow and continuous compounding, equals the sensitivity to the nominal discount rate. They decompose the nominal discount rate into two components, expected inflation and the real discount rate, and implicitly assume that fluctuations in these variables are the causes of a change in the 'theoretical price' of equities. Changes in inflationary expectations or real interest rates are assumed to affect dividend growth rates, thus limiting the effect of a change in interest rates on equity values.

Leibowitz *et al.* derive the formula

$$\frac{\mathrm{d}V_0}{V_0} = -\frac{1}{k-g^*}\left(1-\gamma+\frac{\partial h}{\partial r}\right)\mathrm{d}r - \frac{1}{k-g^*}\left(1-\lambda+\frac{\partial h}{\partial I}\right)\mathrm{d}I, \tag{3.6}$$

where

V_0 is the theoretical price of the stock,
k is the nominal discount rate $= i + h(I, r, ...)$,
i is the nominal risk-free interest rate, equal to $r + I$,
r is a real component of the nominal risk-free interest rate,
I is an inflation component of the nominal risk-free interest rate,
h is the equity market risk premium,
g^* is the nominal growth rate, equal to $g_0 + \gamma r + \lambda I$,
γ is the sensitivity of the growth rate to real interest rates,
λ is an inflation flow-through parameter.

The main advantages of our model (equation (3.5)) over that of Leibowitz *et al.* (equation (3.6)) are that our model (a) is formulated entirely in real terms, (b) has the flexibility to incorporate the impact of changes in inflationary expectations on changes in expectations of real dividend growth and the real discount rate, and (c) includes explicitly a term giving the effects of changes in the real dividend growth rate on equity values. The main disadvantage of our model is the absence of any separation of the risk-free real interest rate and the risk premium, although this could be incorporated as an extension to the model.

Sensitivity measures and observable market data

One use of sensitivity measures is to anticipate the effects of a change in economic variables on equity prices. In particular, a change in the yield on index-linked gilts should, according to our model, affect equity values. It is interesting to note therefore that in the Wilkie (1995a) stochastic investment model, there is no term representing a relationhip between the yields on index-linked gilts and equity dividend yields, although there is a term representing a relationship between the yields on index-linked gilts and on property. Wilkie found that a term linking the yields on index-linked gilts and on equities was not significant, using data for the last 14 years. Preliminary time-series work by Golias (1995), however, indicates that such a relationship has been getting stronger in recent years as the amount of index-linked stock available in the market has increased. This is an interesting area for further empirical research.

4. The importance of dividend yield

Note that, if we consider the aggregate of investors and assume a market that is efficient under rational expectations—so that the present value of future dividends is equal to the market price P—then

$$\frac{\mathrm{d}P}{P} = \frac{\mathrm{d}g - \mathrm{d}j}{j-g},$$

or

$$\frac{dP}{P} = \frac{dg - dj}{\text{dividend yield}}, \tag{4.1}$$

because the dividend yield equals $j - g$. Equation (4.1) together with equations (3.2) and (3.4), imply that low-yield shares (i.e. those with a low dividend yield) should have greater sensitivity to changes in g and j than high-yield shares. This is consistent with the view often expressed by practitioners that 'high-yield shares are defensive'.

It should be noted that actual share price volatility (as measured by standard deviation of return) is also related to the uncertainty surrounding both the expected real dividend growth rate and the market's real discount rate. A share with a low dividend yield could therefore be less volatile than one with a higher dividend yield if the market's estimates of real dividend growth for the latter tended to vary over time more than those for the former. But the classically volatile stock is one with a high expected real dividend growth rate, a high degree of uncertainty concerning the dividend growth projections, and in an economy with low risk-free real rates of return. The transition from the perfect-certainty model of this paper to a model which accommodates uncertain future dividends and uncertain discount rates is an area for further research.

Our discussion on sensitivity measures above shows that low-yield shares have greater sensitivity to changes in the underlying fundamental real variables than high-yield shares. This suggests that low-yield shares should generally offer higher expected returns to reflect the higher risk premium investors would demand. However, other factors such as taxation and market inefficiency may play a part. Indeed, empirical evidence both in the US (e.g. Elton *et al.* 1983) and in the UK (e.g. Levis 1989; Fisher & Bradford 1995) shows that low-yield shares have historically given *lower* gross returns than high-yield shares.

5. Conclusion

Using a model incorporating both a short-term real dividend growth rate and a long-term real dividend growth rate, we derived an expression for the present value of future dividends for an equity share and hence derived sensitivity measures for an equity share. These sensitivity measures can be useful in assessing the risk of equity share investments. In particular, we found that low-yield shares tend to be more sensitive to changes in both the real discount rate and the estimated long-term real dividend growth rate, compared with high-yield shares.

Restricting the discussion to a single real growth rate (i.e. with the short-term and long-term rates equal), we then compared our model, which is formulated entirely in real terms, with that of Leibowitz *et al.* for equity duration. Particular strengths of our model are the flexibility to incorporate the impact of changes in inflationary expectations on the fundamental real variables, and the inclusion of a term to allow for changes in expectations of real dividend growth rates.

One might expect that low-yield shares should offer higher expected returns than high-yield shares to compensate for their greater sensitivity (which is in some sense a measure of risk). In fact, empirical research both in the UK and US suggests that low-yield shares give *lower* gross returns than high-yield shares.

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